The Concept of Quadratic Functions - ACTIVITIES

Learning Objectives

LO1: Students learn to calculate values of y for different values of x using quadratic functions

LO2: Students learn how to tabulate and plot quadratic functions

LO3: Students learn that many economic relationships are rarely linear and some can be expressed using a quadratic expression.
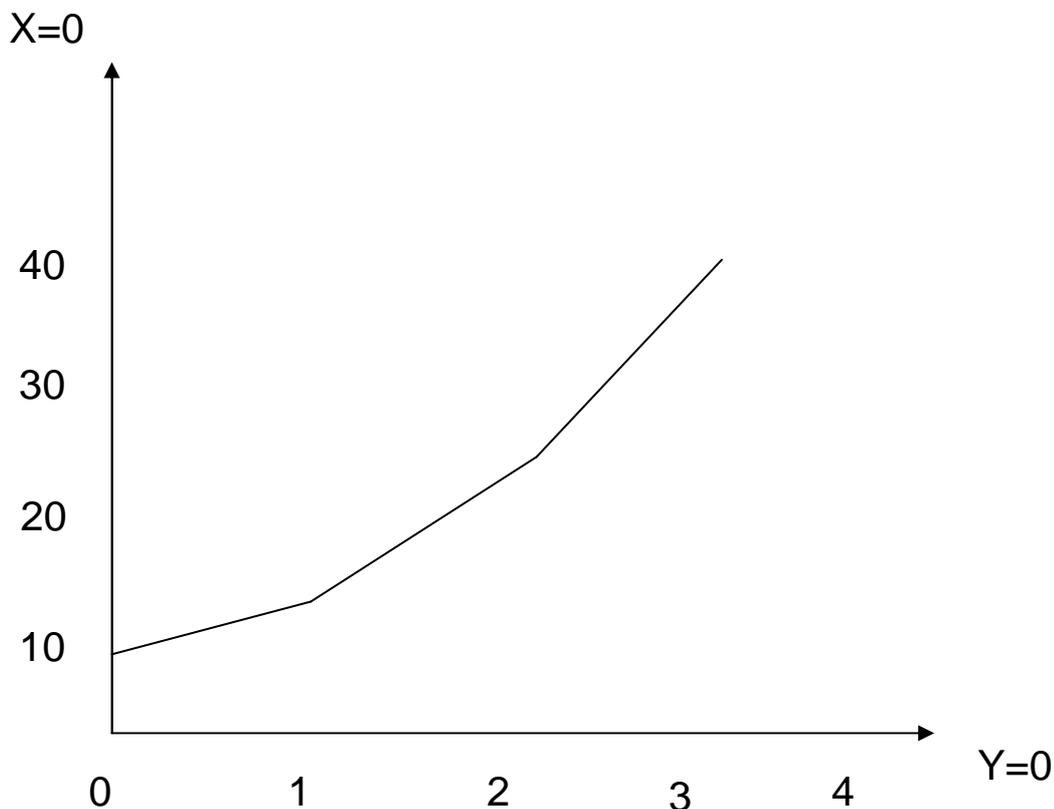
Task One

To begin with, let us assume that we are in a small group setting, such as a tutorial or problems class, and we will show later how the activity can be adjusted for the large group (lecture theatre) setting.

The first stage is to split the group into pairs or threes, which can be done with the people sitting next to each other – there is no need to enforce randomness on this procedure as in fact that might be threatening for some students and again create barriers to learning. The second stage is to provide each group with a piece of graph paper with a relatively simple non-linear function written onto it (e.g. $y = 10 + 4x^2$).

Each group could have the same function or you could vary them so that there is exposure to a number of equations. Another possibility is for the students to be given a series of equations and they chose one of them to examine, which puts the element of control into their hands, again helping to create a more positive atmosphere around the learning.

The exercise they are then set is to calculate the change in the dependent variable (y) for the same (or possibly different) changes in the independent variable (x). These can be written down in tabular form on the sheet and then they can be plotted. Thus for example, for $y = 10 + 4x^2$ it might look like this:

(x)	(y)
0	10
1	14
2	26
3	46

The outcome of this exercise is that it should be apparent that for a given move from one value in (x) the change in (y) varies and thus the previously simple relationship between (x) and (y) found in linear equations no longer holds. Crucially too the impact of changes in (x) on (y) depends on where you start with (x) – a low value or a high value for instance.
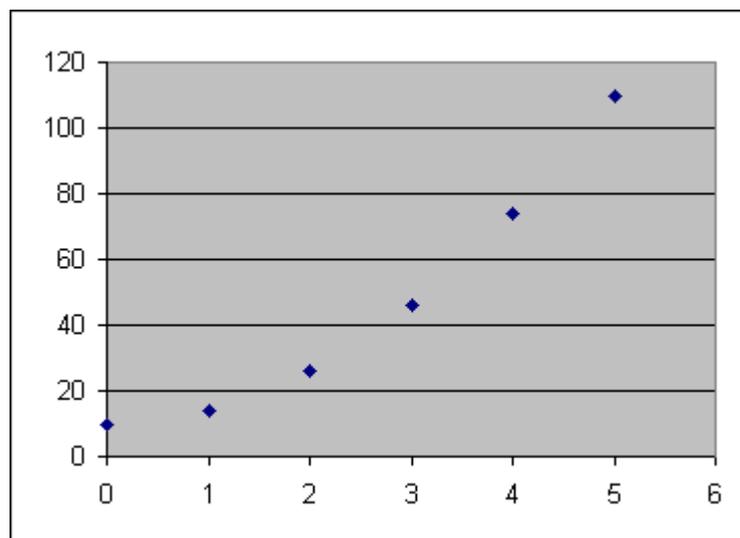
Another issue here of course is how the lines are plotted. At first it is best to make explicit that you simply want to plot the points as straight lines between two pairs of co-ordinates. Once this has been done then, discussion of what happens to all the values in between the whole numbers we have chosen and that can lead to plotting a smooth curve. This reinforces the staging process from linear models to non-linear as the shape becomes exacerbated.

In essence, the exercise builds on prior learning as it requires manipulation of two variables to find solutions to an equation and then plotting these on a graph, both of which they will have done previously. In a large group setting the sheets can be handed out at the start of the session and students can work on their own. Values for x are provided and they have to find y. They then offer their answers to the lecturer who has a plot on the whiteboard or computer at the front of the lecture theatre. As answers are plotted the non-linear nature of the relationship should become apparent quite readily. This can be done using Excel and plotting from the data therein which could be pre-loaded with more data points than they provide in responses but which allow for the curvature of equations to be emphasised.

An Example of an Excel chart with Data:

y= 10 + 4x2

x	y
0	10
1	14
2	26
3	46
4	74
5	110

Task Two

Given the demand function $Q_D = 75 - (1/4)P$

    a. Find the total revenue function written in terms of Q [TR = $P(Q)$]

    b. Calculate the point at which total revenue is equal to zero

    c. Calculate the point at which total revenue is maximised.

Task Three

A manufacturer faces two types of costs in its production process, fixed costs which are equal to £1000 and variable costs which are equal to £2 for each tyre produced.

  a. State the total cost function for this firm

  b. Calculate average costs

  c. Calculate total costs if 700 tyres are produced.

Task Four

Using the total revenue function calculated from question 1 and the total cost function calculated in question 2.

  a. Derive the profit function for the firm

  b. What is the value of profits when production equals 125 units?

Task Five

A company discovers the following economic information about its costs and demand function:

Demand Data	*Cost Data*
$Q = 33 - \dfrac{1}{2}p$	Fixed costs are £200
	Variable costs are £8 per unit

  i) Derive the profits function for the firm

  ii) What is the breakeven quantity?

  iii) At what output level would profits be maximised?

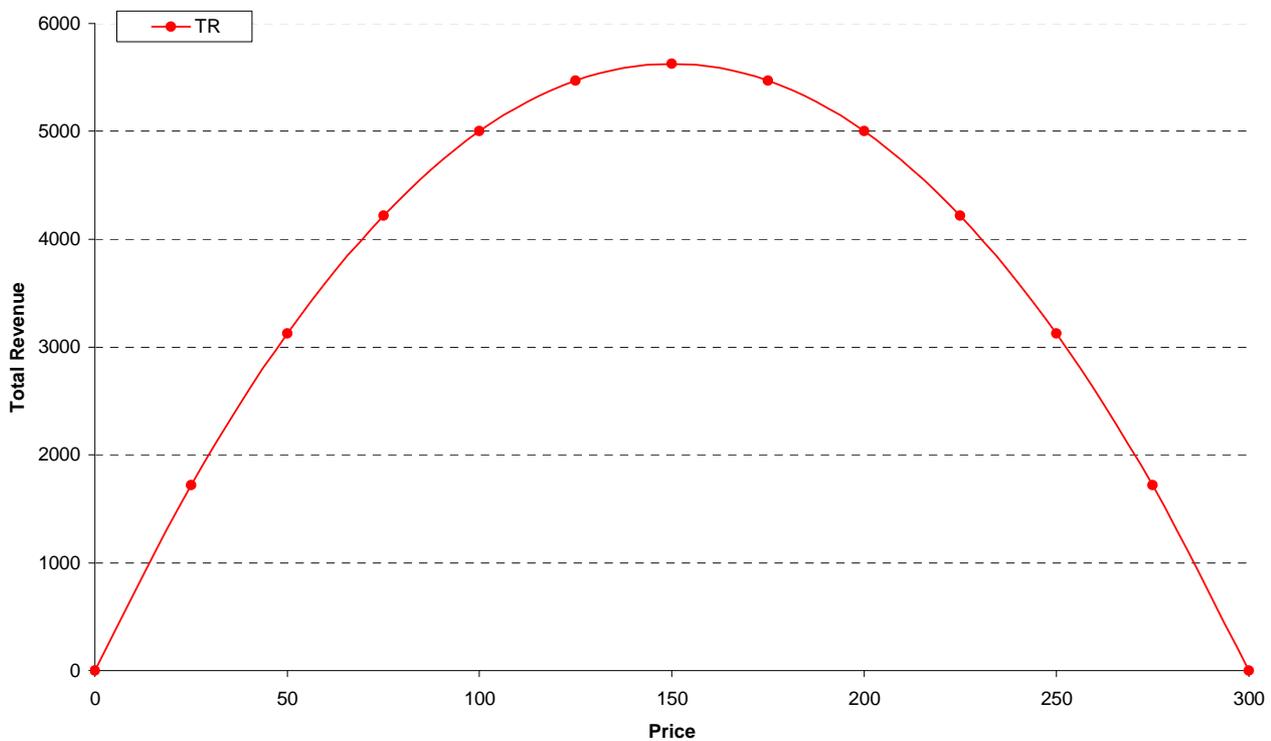# The Concept of Quadratic Functions - ANSWERS

Task One

See Task

Task Two

a. $TR = 75p - \frac{1}{4}P^2$

b. TR=0, p=300

c. TR is maximised when P=150 (see diagram below)

Task Three

Let t denote the number of tyres produced

a.　　TC = 1000 + 2t

b.　　AC = TC/t = $\frac{1000}{t} + 2$

c.　　TC=£2400 when t=700

Task Four

a.   Profit = TR-TC = $300Q - \dfrac{1}{4}Q^2 - 1000 - 2Q$

b.   Profit = **-£26,250** ie. a loss

Task Five

i)

$$Q = 33 - \frac{1}{2}p$$
$$\Rightarrow p = 66 - 2Q$$
$$TR = p \times Q = 66Q - 2Q^2$$
$$\pi = TR - TC = 66Q - 2Q^2 - 200 - 8Q$$
$$\pi = -2Q^2 + 58Q - 200$$

ii) Breakeven occurs when π=0

$$\pi = -2Q^2 + 58Q - 200 = 0$$

Factorising we get:

$$(-2Q + 50)(Q - 4) = 0$$

Q=4 or Q=25

So, the firm breaks even when output is 4 units and also when output is 25 units

iii) Profits maximised $\dfrac{\partial \pi}{\partial Q} = 0$

$$\pi = -2Q^2 + 58Q - 200$$

$$\frac{\partial \pi}{\partial Q} = -4Q + 58$$

$$\frac{\partial \pi}{\partial Q} = 0$$

$$-4Q + 58 = 0$$

$$Q = \frac{58}{4} = 14.5 units$$

Cubic and other polynomial functions - ACTIVITIES

Task One: Battleships

Despite the obvious benefit of sheets of practice questions for promoting understanding and exam preparation by students it can often be the case that this preparation is not done, in particular if problem classes simply work through the answers. Providing an incentive to students to complete the tutorial sheets is therefore of use.

In battleships, students are split into teams and given a reasonably long list of questions that they must work through, as well as their battleship grid and battleships at the end of the previous tutorial. In time for the next tutorial it is the task for each team to work through the questions on the list in their teams and decide where they will locate their battleships. The game is played as normal battleships but to get a go at hitting the opponents ships they must have answered a question correctly. It can help to call question numbers from the list randomly, even if questions are structured to form part of a *bigger* questions, so there is a mix between hard and easy questions. Students can get very involved with such a game.

Task Two: Leontief Production Functions

Students do sometimes fail to recognise that production functions are not sacrosanct and that there are many versions of production function that firms can apparently work from. Indeed, the very fact that different forms exist of the same type of function is often a revelation to some students! So the notion that there may in fact be non-linearities in production functions is probably a very big step for some students to take in terms of their comprehension of the links between maths and economics.

Students are asked to consider the Leontief case which has the virtue of being linear around a break-point that makes the whole function non-linear. It is an extreme case of production function but does illustrate the issue of non-linearity quite nicely. To do this, you could get the students to consider themselves as the local council which is charged with keeping streets clean. They are a low-tech council and rely on two factors of production; workers and brooms. The discussion can then centre on how best to combine workers and brooms to sweep the streets of whichever district you care to choose – maybe a very messy student one!

Using either graph paper or indeed spreadsheets in Excel the students can draw up simple table of output (i.e. clean streets) in relation to inputs on the basis that one worker with one broom creates two clean streets for example. In this way you can then show how everything is constrained by the factor that is least abundant.

Thus we have 20 brooms but only 6 workers we only achieve output of 12 clean streets which is the output associated with 6 workers and so forth. As a change, you could then suggest that the council now invests in new technology by buying a machine to replace the brooms and the output relationship now is one worker plus one machine equals 6 clean streets. Before repeating the exercise, see what the students expect to happen to the isoquants and then explain the outcome they find.

While this does not give us the curved non-linearity of many of the other exercises in this area it does highlight how production functions generate some unusually shaped isoquants and the difficulties this provides for finding the optimal combination of factors of production.

Task Four: Production functions, Growth and Economic Development
This topic works well as an extension of the discussion topic on economic convergence described above. The growth of output is explained by the growth of inputs into the production function. The relative importance of these inputs in a Cobb-Douglas production depends upon the value of the elasticity parameters on each of the inputs. As a result the policy can change. This debate is well known in the growth accounting literature and has been discussed in particular with respect to the growth in the South East Asian economies. As in that literature, this debate works best with respect to the share of physical capital investment versus technological progress/productivity in growth because productivity is measured as the residual.

This exercise offers another way of introducing the idea of production functions, output is made up of the inputs into the production function. Assume a production function of the form, $Y = AK^{\alpha}H^{\beta}$, where Y is output, K is physical capital, H is human capital and A is productivity/technology. The basic idea of the exercise is to demonstrate that for a given rate of increase in the inputs their relative importance in describing output varies. A discussion should also be made here about why productivity cannot be measured and is therefore calculated as the residual.

This is most easily done if the production function is in log form so that $\ln A = \ln Y - \alpha \ln K - \beta \ln H$.

Get them initially to work out an initial value of output for a given set of inputs and parameter values, it is easier if they are given the log values of the inputs. They could then work out a final value and be asked what percentage of the change in output was explained by human capital and physical capital and what by productivity.

China	Output per capita	Physical capital α=0.6	Human capital β = 0.4	Productivity
1990	6.4	8.3	2.6	0.38
2006	7.36	9.59	2.91	0.44

Note: All values are logged.

Then you could change the value of the parameters and repeat the exercise, say decreasing the alpha parameter to 0.4. This sometimes works best if one of these iterations is done collectively and they are them broken into small groups and asked to repeat the exercise for the new parameter set (different parameters could be given to different groups). A discussion could then be held as to which input, human capital, physical capital or productivity governments should focus on.

Reference could be made here to the recent UK Budget which discuss these issues. This could be finished off with a discussion how diminishing marginal returns would mean we would not want to accumulate one input alone. While it is helpful to use the logged version of the function it is important that the students are reminded of what the normal Cobb-Douglas production function looks like during this task.