Arithmetic operations - ACTIVITIES

ACTIVITY ONE

<u>Learning Objectives</u>

LO1. Students to consolidate meaning of arithmetic operators

LO2. Students to learn how to confidently use arithmetic operators

Students are put into small groups and given laminated cards.

Three laminated cards for each number 0 -9 inclusive. Two laminated cards for each arithmetic operator +, -, x, ÷. Students given 3 laminated cards with opening bracket '(' , 3 laminated cards with closing bracket ')' and 1 '='.

The task is to agree how to arrive at given numbers using the laminated cards.

For example,

Students have: 000,111,222,333,444,555,666,777,888,999,(((,))), ++,--,xx,÷÷,=

Task 1 : Use two mathematical operators to arrive at 73

Task 2: Use all four operators to arrive at an even double digit number

ACTIVITY TWO

<u>Learning Objectives</u>

LO1. Students to consolidate basic meaning of arithmetic operators

LO2. Students to learn how to confidently use arithmetic operators

Task One

Students to complete the missing gaps from the following expressions

i) ………………….. x ……………… = 225

ii) ………………… ÷ ……………… = 64

iii) (……….. + ……….) / 10 = 200

iv) (………… x ……….) / (……..+ 7……) = 200

v) (………… ÷……….) x (6 + …….) = 800

Task Two

Which statements are TRUE and which are FALSE?

	Tick (✓)	
	True	**False**
(a/b) = a x (1-b)		
4 + 6/5 =2		
6 + 6/2 = (6+6)/2		
13 **…….** 5 = 16.25 × **…….** The two missing gaps are ‘x’ and ‘5’ respectively		

Task Three

Complete the table below. The first row has been done for you.

Figure	**Operator** **(+, -, ×, ÷)**	**Figure**	**Equals**	**Answer**
5	×	31	=	155
	+	17	=	125
	÷	57.5	=	230
-4	-		=	-10
13	÷		=	
16			=	80
	÷		=	169

169	+		=	150
346	÷		=	173
567		383		

Task Four (Extension Task)
The following data is obtained from a large database of company financial information. Each company produces only one product.

	Sales (£m)	Profits (£m)	Price of product (£)	Quantity sold (millions of units)
Schumacher Ltd		45	4	25
Coulthard Plc		125	4	62.5
Wurz and Co		25	2	87.5
Alonso Plc		25	2	112.5
Massa and Co		15	12.5	24
Raikonnen		60	5	63
Montoya Ltd		10	2.5	20

Part 1
Sales are calculated by multiplying price by quantity sold.
Complete the sales column using the simple formula: Sales = Price x Quantity

Part 2
Calculate:
(a) total sales of the seven companies;
(b) total profits earned by the seven companies;
(c) total profits earned by the three most profitable companies

Part 3
Express a formula which would calculate average profit per unit sold

Part 4
(a) Which company has the highest profit per unit sold?
(b) What is the profit per unit of this company?

Part 5
A member of the research team thinks that profits should be compared against sales and decides to work out the gross profit to sales ratio. This is calculated by dividing profits by sales. Which company has the highest profit to sales ratio?

Part 6
What is the lowest profit to sales ratio?

Arithmetic operations - ANSWERS

ACTIVITY ONE

There are a large number of solutions and, of course, the purpose is not to create a particular solution but rather to practise the confident use of arithmetic operators.

<u>Possible solutions</u>

Task 1: 100 – 30 + 3 = 73

(5 x 3) + 58 = 73

(75 ÷5) + 58 = 73

Task 2: ((10 x 12) ÷(4 x 3)) + 34 - 26= 18

((17 + 3) x (28 – 10))÷9 = 40

ACTIVITY TWO

Task One

i) 15 x 15 =225, 5 x 45 =225, 9 x 25 = 255 etc

ii) 128÷2 = 64, 256÷4 = 64, 4096÷64=64 etc

iii) (1700+300)/10 = 200, (1000+1000)/10= 200, (500+1500)/10 = 200 etc

iv) (50 x 40)/(3+7) = 200, (140 x 20)/ (7+7) =200, (70 x 80)/(7+21) = 200 etc

v) (160 ÷2) x (6+4)=800, (150÷3) x (6 + 10) =800 etc

Task Two

	Tick (✓)	
	True	**False**
(a/b) = a x (1/b)	✓	
4 + 6/5 =2		✓
6 + 6/2 = (6+6)/2		✓

If: 13 ……. 5 = 16.25 x ……. The two missing gaps are ‘x’ and ‘4’ respectively	✓	

Task Three

Figure	Operator (+, -, ×, ÷)	Figure	Equals	Answer
5	×	31	=	155
108	+	17	=	125
13225	÷	57.5	=	230
-4	-	**6**	=	-10
13	÷	**2**	=	**6.5**
16	×	**5**	=	80
676	÷	**4**	=	169
169	+	**-19**	=	150
346	÷	**2**	=	173
567	**+**	383	**=**	**950**

Task Four (Extension Task)
Part 1

	Sales (£m)
Schumacher Ltd	100
Coulthard Plc	250
Wurz and Co	175
Alonso Plc	225
Massa and Co	300
Raikonnen	315
Montoya Ltd	50

Part 2

(a) £1415m
(b) 305m
(c) £230m

Part 3

Average profit = Total profit/ quantity sold

Part 4

(a) Coulthard plc
(b) £2.00 per unit

Part 5

Coulthard plc (profit to sales ratio is 0.5 or 50%)

Part 6

The lowest ratio is that of Massa and Co with a ratio of 0.05 or 5%

Fractions - ACTIVITIES

ACTIVITY ONE

LO1: Students learn how to calculate simple fractions

Task One
Complete the database below using this information.

	Price (£ per kilo)	Cost of 3 kilos	Cost of 3 kilos if price is halved
Apples	2.00		
Bananas	3.50		
Cherries	4.00		
Dates	6.00		
Figs	2.45		
Gooseberries	1.25		
Kiwi	2.90		
Loganberry	3.50		
Plums	4.15		
Raspberries	5.00		
Strawberries	6.30		
Watermelon	1.50		

Task Two
How much would 6 kilos of dates cost using the original prices?

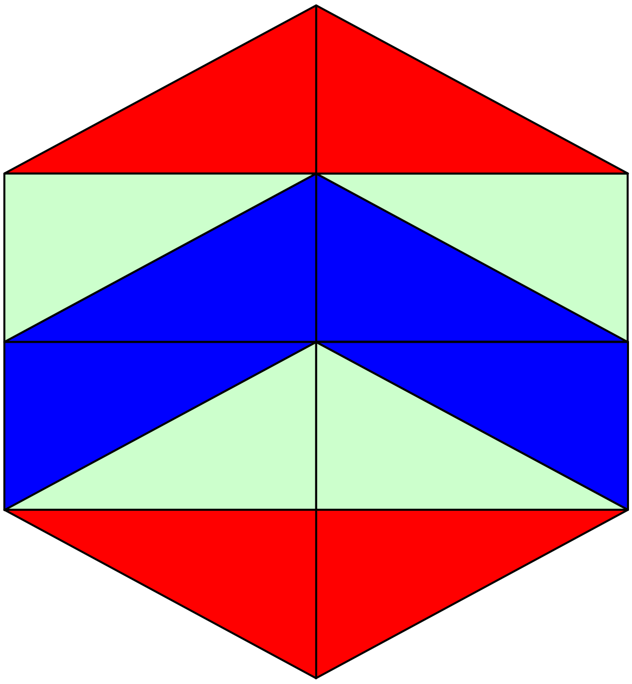
Task Three
If the price of apples halves, what is the difference in total cost for 3 kilos of apples?

Task Four
Express the cost of 3 kilos of apples as a fraction of the cost of 3 kilos of dates. Use the original prices.

ACTIVITY TWO

Learning Objectives
LO1: Students learn how to calculate simple fractions
LO2: Students learn how to apply fractions to simple probability tasks

The owner of a large fairground is considering whether to introduce a new game. The game is essentially a variation of darts. The board is made of 12 identically sized triangles and looks like this:



Each triangle is right angled and has a base of 10cm and a height of 6cm.

Task One
Calculate the total area (in cm^2) of:
(a) all of the blue triangles; and
(b) the area of the board.

Task Two
What fraction of the board is red?

Task Three
If a contestant hits a red or blue area he wins £5.
(a) if a contestant successfully hits the board, what is the probability that they will win £5?
(b) if, on average, a contestant hits the board once in two attempts, what is the likelihood they will hit a blue area on their first attempt?

Fractions – ANSWERS

ACTIVITY ONE

Task One

	Price (£ per kilo)	Cost of 3 kilos	Cost of 3 kilos if price is halved
Apples	2.00	6.00	3
Bananas	3.50	10.50	5.25
Cherries	4.00	12.00	6
Dates	6.00	18.00	9
Figs	2.45	7.35	3.675
Gooseberries	1.25	3.75	1.875
Kiwi	2.90	8.70	4.35
Loganberry	3.50	10.50	5.25
Plums	4.15	12.45	6.225
Raspberries	5.00	15.00	7.5
Strawberries	6.30	18.90	9.45
Watermelon	1.50	4.50	2.25

Task Two
£36.00

Task Three
The apples cost £3.00 less.

Task Four
¼

ACTIVITY TWO

Task One
(a)120cm^2
(b) 360cm^2

Task Two
1/3

Task Three
(a) 8/12 or 2/3
(b) ½ x 1/3 = 1/6

Percentages - ACTIVITIES

ACTIVITY ONE

Learning Objectives

1) Students understand the mapping between fractions and percentages
2) Students can independently calculate simple percentages
3) Students can apply percentages to a given problem

Consider the table below. Draw a line between each fraction and the equivalent percentage.

Fraction
1/10
½
1/3
1/4
1/6
1/7
2/3
4/6
5/7
9/10
15/20
7/14
23/100

Percentage
71.4%
50%
25%
66.6%
16.6%
50%
14.2%
90%
75%
33.3%
23%
10%
66.6%

ACTIVITY TWO

This activity could be preceded by the video clip (Field 1.E.2) which sets a strong and applied macroeconomic tone.

Learning Objectives

1) Students to understand basic macroeconomic data
2) Student to be able to calculate percentages and relate numerical data to graphical illustrations

An economist collates some basic data on a number of countries. This is summarised in the tables below for two years 1995 and 2005.

Data for 1995

	Unemployment (millions)	Inflation (% per annum)	Population (millions)	GDP (£ billions)
Country A	5	1	125	2125
Country B	3	3	100	1250
Country C	2	2	57	1311
Country D	13	4	250	5000
Country E	6	1	165	2640

Data for 2005

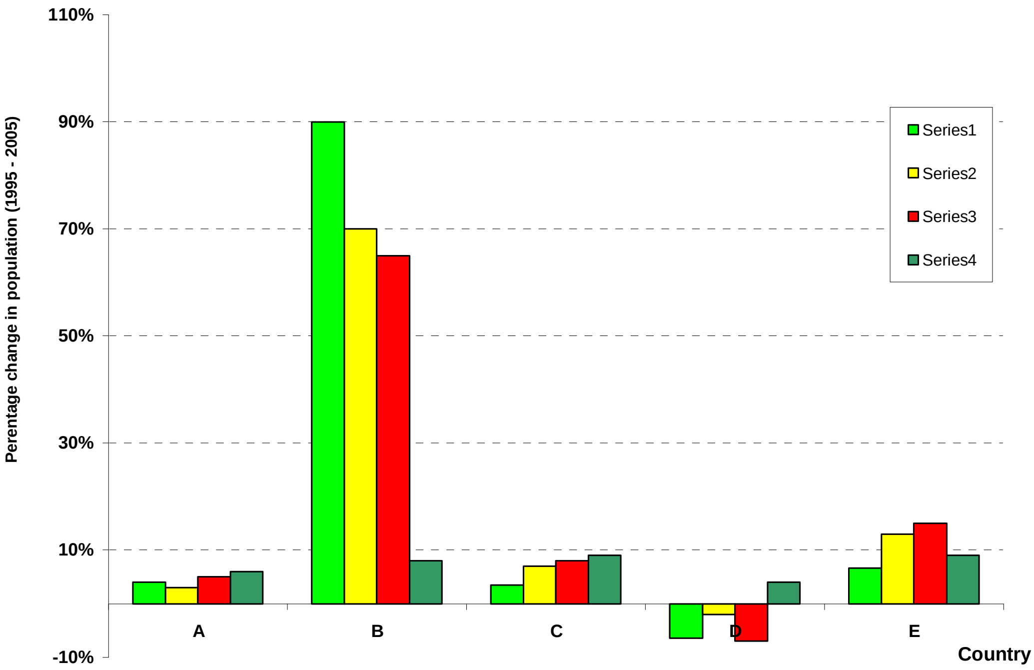
	Unemployment (millions)	Inflation (% per annum)	Population (millions)	GDP (£ billions)

Country A	4	2	130	2125
Country B	3	4	190	1250
Country C	3	1	59	1311
Country D	12	6	234	5000
Country E	8	3	176	2640

Task 1
(a) Which country experienced the biggest positive change in unemployment?
(b) What was the percentage change?

Task 2
Which country had the biggest unemployment rate (unemployment as a proportion of the total population) in 1995 and what was the figure?

Task 3
Look at the graph below. Which series (Series 1,2,3 or 4) illustrates the percentage changes in population (1995 to 2005)?



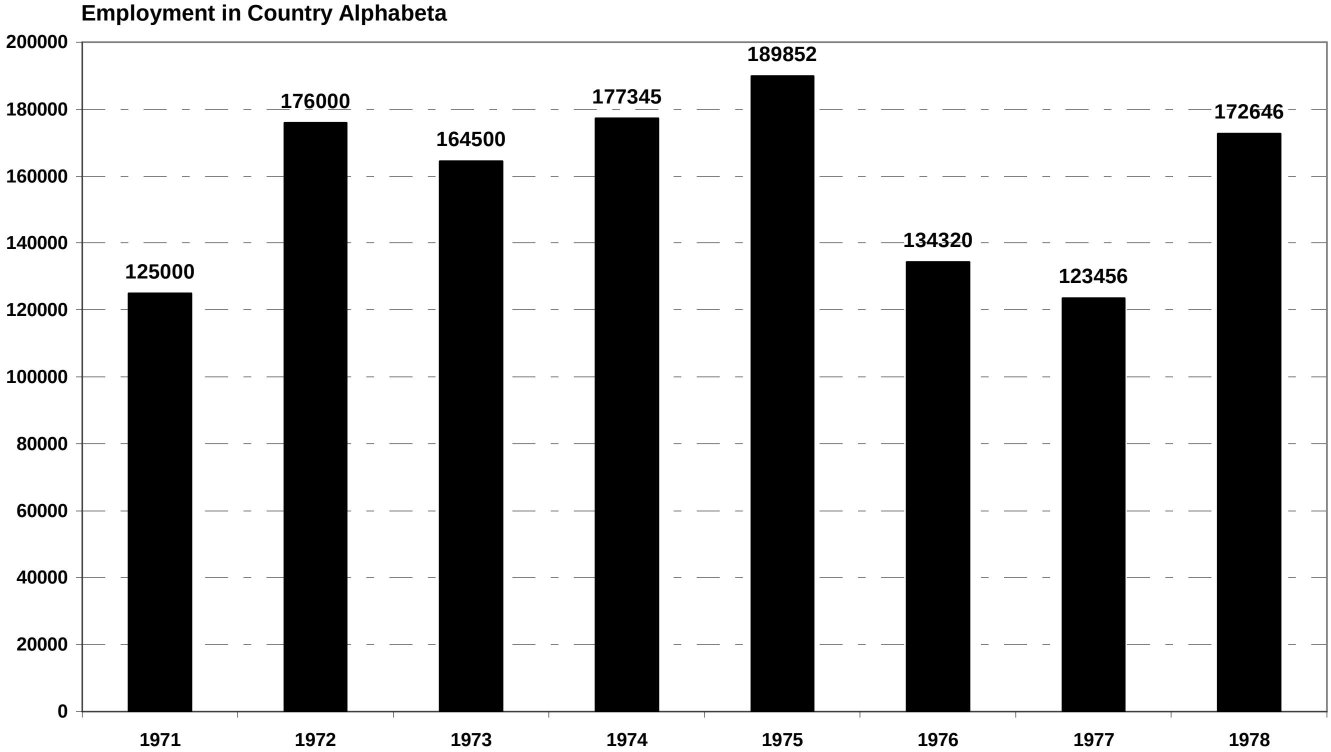
ACTIVITY 3

__Learning objectives__

LO1. Students to be able to calculate simple percentages
LO2. Students to be able to calculate changes in percentages, and make simple inferences.
LO3. Students to be able to use percentages in simple "What..If" scenarios

An economist researches Country Alphabeta and finds that employment in the country during the 1970s varied considerably from one year to the next.

He chooses to show his data graphically in a brief research paper. His graph is shown below.



He sends his paper to a colleague for comments. His friend says that the graph is difficult to interpret and poses a number of questions. Your task is to answer these on behalf of the researcher.

Question 1
Using the above data produce a table showing the percentage change in employment year on year?

Question 2

(a) Calculate the change in employment in terms of the number of people employed from the start point in 1971 to the end point in 1978

(b) What is the average percentage change in employment from 1971 to 1978?

(c) What would employment be in 1979, 1980 and 1981 if the average percentage change calculated in (b) continued?

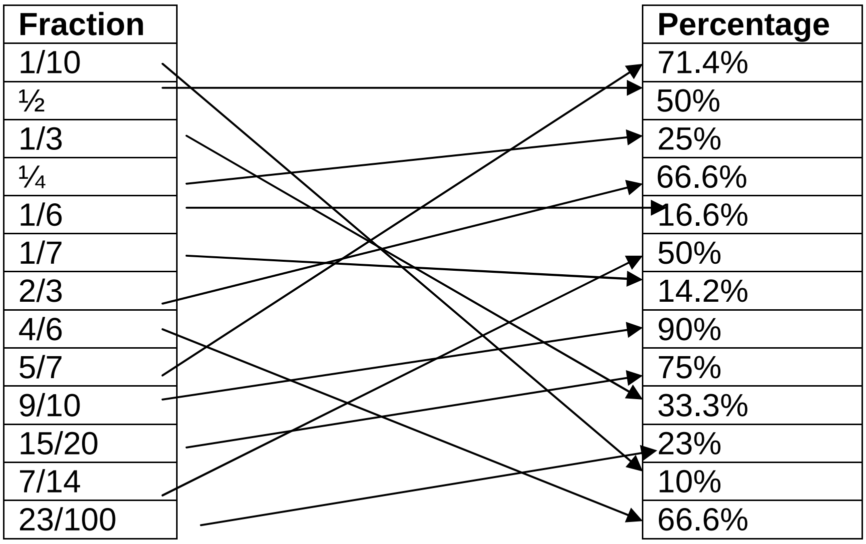
Question 3
The researcher is told that his research is incorrect and that in fact:

i) employment was indeed 125,000 in 1971; and
ii) employment rose each year by exactly 2.5%.

What would the employment figure for 1978 be?

Percentages - ANSWERS

ACTIVITY ONE



ACTIVITY TWO

Task 1

Country C with 50% increase in unemployment

Task 2

Country D with 5.2%.

Task 3

Series 1(green bars)

ACTIVITY THREE

Question1

1971	-
1972	+40.8%
1973	-6.5%
1974	+7.8%
1975	+7.1%
1976	-29.3%
1977	-8.1%
1978	+39.8%

Question 2

(a) =172646 – 125000= **47646**

(b) =((172646-125000)/125000)* 100% = **+38.1%**

(c)

Year	Employment
1971	125000
1972	176000
1973	164500
1974	177345
1975	189852
1976	134320
1977	123456
1978	172646
1979	**238424**
1980	**329264**
1981	**454713**

Question 3

Employment figure would be 125000 x $(1.025)^7$ = 148586

Powers - ACTIVITIES

ACTIVITY ONE

Learning Objectives

LO1 : Students learn how to use simple calculations using powers

LO2 : Students learn how to manipulate expressions involving powers

Task One

Work out the following expressions using simple powers:

Expression	Answer
3^3	
5^3	
6^4	
6^6	
7^3	
11^3	
4.5^3	
2.2^2	
$4.4^{4.4}$	

Task Two

In some way, powers or indices are similar to multiplication and division.

One number (A) may be divided by another number (B) to calculate an answer (C). Alternatively, A may be multiplied by the reciprocal of B to calculate C

Put another way,

$A \div B \equiv A \times 1/B = C$

Similarly, powers can be used in a ‘symmetrical way’. A negative power can be expressed as a positive power when it is the reciprocal

$3^{-2} = 1/3^2 = 1/9$

$4^{-3} = 1/4^3 = 1/(4x4x4) = 1/64$

$5^{-5} = 1/5^5 = 1/(5x5x5x5x5) = 1/3125$

Using this knowledge, answer the following questions:

Expression	**Tick one of the columns**	
	True (✓)	**False (✓)**
$2^2 = 4$		
$2^2 = -4$		
$3^3 = 9$		
$3^4 = 18$		
$3^{-2} = 1/9$		
$4^{-2} = 1/16$		
$10^2 = 4.64^3$		
$9^2 = 4^3$		
$12^3 = 36$		
$8^3 = 2^9 = 2 \times 16^2$		
$6561 = 81^{-2}$		
$4^{-2} = 16^{-1}$		
$12^{-3} = 2^{-2} \times 432^{-1}$		

Task Three

An economist creates an expression – known as a “production function” which describes how capital (K) and labour (L) can be combined to create output (Q).

If:

$Q=K^{\alpha}L^{(1-\alpha)}$

(a) Complete the table:

α	K	L	Q
0.1	0	100	
0.1	10	90	
0.1	20	80	
0.1	30	70	
0.1	40	60	
0.1	50	50	
0.1	60	40	
0.1	70	30	
0.1	80	20	
0.1	90	10	
0.1	100	0	

(b) What do you notice about the powers of α and β?

(c) If:

$Q= K^{\alpha}$ then complete the tables:

	α	Q		α	Q		α	Q
	0	1		0	+1		0	1
	1	5		1	-1		1	0.1
K= 5	2	25	**K=-1**	2	+1	**K=0.1**	2	0.01
	3	125		3	-1		4	0.0001
	4	625		4	+1			
	5	3125		5	-1			
	6	15625		6	+1			

(d) In each case, rearrange the expression to make 'α' the subject:

i) $Q=K^{\alpha}$

ii) $3Q=4K^{\alpha}$

iii) $3K= 2Q^{\alpha}$

iv) $3\alpha^2=Q/K$

ACTIVITY TWO

Learning Objectives

LO1 : Students learn how to use indices to calculate simple compound interest

Task One

Jean Cheesman invests £10,000 in an interest bearing account. The bank will pay her 3% on the balance she has in the account at the end of the year.

What simple expression would calculate her:

(a) bank balance(B) after n years [assumes she withdraws none of her money]?

(b) the interest she receives after n years

(c) the **real** value of her interest if inflation is always two thirds of the rate of interest?

(d) What would the rate of interest need to be if Jean Cheesman expected her savings balance to be £17,000 after 6 years?

(e) How many years, to the nearest whole year, would Jean Cheesman need to invest her money if:

- her principal sum was still £10000;
- the rate of interest was 2.3%; and
- she wanted to have at least £19,750?

Powers - ANSWERS

ACTIVITY ONE

Task One

Expression	Answer
3^3	27
5^3	125
6^4	1296
6^6	46656
7^3	343
11^3	1331
4.5^3	91.13
2.2^2	4.84
$4.4^{4.4}$	677.94

Task Two

Expression	Tick one of the columns	
	True (✓)	False (✓)
$2^2 = 4$	✓	
$2^2 = -4$		✓ $2^2 = 4$
$3^3 = 9$		✓ $3^3 = 27$
$3^4 = 18$		✓ $3^4 = 81$
$3^{-2} = 1/9$	✓	
$4^{-2} = 1/16$	✓	
$10^2 = 4.64158^3$	✓	

$9^2 = 4^3$		✓ $9^2 = 81$ $4^3 = 64$
$12^3 = 36$		✓ $12^3=1728$
$8^3 = 2^9 = 2 \times 16^2$	✓	
$6561^{-1} = 81^{-2}$	✓	
$4^{-2} = 16^{-1}$	✓	
$12^{-3} = 2^{-2} \times 432^{-1}$	✓	
$6^{-6} \times 3^2 \times 4^2 = 1/300$		✓ $6^{-6} \times 3^2 \times 4^2 = 1/324$

Task Three

(a)

α	K	L	Q
0.1	0	100	0.00
0.1	10	90	72.25
0.1	20	80	69.64
0.1	30	70	64.31
0.1	40	60	57.62
0.1	50	50	50.00
0.1	60	40	41.66
0.1	70	30	32.65
0.1	80	20	22.97
0.1	90	10	12.46
0.1	100	0	0.00

(b) The powers or indices of α and β sum to 1. This particular production function is a special case: the Cobb-Douglas function.

(c)

	α	**Q**		**α**	**Q**		**α**	**Q**
	0	1		0	+1		0	1
	1	5		1	-1		1	0.1
	2	25		2	+1		2	0.01
K= 5	3	125	**K=-1**	3	-1	**K=0.1**	4	0.0001
	4	625		4	+1			
	5	3125		5	-1			
	6	15625		6	+1			

(d) i) $\alpha = \ln Q / \ln K$

ii) $\alpha = \ln 3Q / \ln 4K$

iii) $\alpha - \ln 3K / \ln 2Q$

iv) $\alpha = \sqrt{(Q/3K)}$

ACTIVITY TWO
Task One

(a) $B = 10{,}000 \times 1.03^n$

(b) $i = (B - 10{,}000)$ or, $10000(1 - 1.03^n)$

(c) Real $i = 10000(1 - 1.01^n)$

(d) $i = \sqrt[6]{(1.7)} = 1.09$ i.e. 9% per year for each year

(e) $n = \ln(19750/1000)/\ln(1.023) = 29.93$ years or 30 years to the nearest whole year.

Logarithms - ACTIVITIES

ACTIVITY ONE

Learning Objectives

LO1: Students to understand the meaning of logarithms and understand rules

LO2: Students to be able to confidently manipulate simple expressions with logarithms

Task One

An economist discovers a link between inflation (I) and employment (E) which she believes to be:

$I = \ln(E^{\alpha})$

Complete the table and graph the relationship

α	E	E^{α}	$I = \ln(E^{\alpha})$
2	1		
2	1.1		
2	1.2		
2	1.3		
2	1.4		
2	1.5		
2	1.6		
2	1.7		
2	1.8		
2	1.9		
2	2		
2	2.1		
2	2.2		
2	2.3		
2	2.4		
2	2.5		
2	2.6		
2	2.7		
2	2.8		
2	2.9		
2	3		

Task Two

If: $I = 50 \times \alpha^{\beta}$ and $\alpha = 2.0$ and $I = 3.0$

What is β?

ACTIVITY TWO

Learning Objectives

LO1: Students to be able to explain the meaning and significance of logarithms

Task One

Complete the missing gaps using the words and phrases provided. There are more words and phrases than you need to correctly complete the gaps so select carefully!

Logarithms in economics

Logarithms are a useful economic tool, which are closely related to powers and **………..**
We know that 16 = $2^{\cdots\cdots}$ where the number **……….** is the ……………… or exponent. It is sometimes also known as the index.
Logarithms are particularly useful when analysing rates of ……………. and …………….. A pharmaceutical company, for example, might want to model rates of growth of …………………. or an economist might be interested to see how ……………. changes over time.
Problems concerning how much interest an investor can expect to receive or how much a sum of money would be worth in …………………. after a period of inflation are essentially issues surrounding ……………………….. These can be easily solved using logarithms.

In a simple logarithmic expression such as $A=B^{\alpha}$ w e can rearrange using logarithms to show that lnA=……..ln…….. where ln is the natural logarithm. A natural logarithm simply means a logarithm to the base ……………. where e is a constant approximately equal to ……..

Table of words

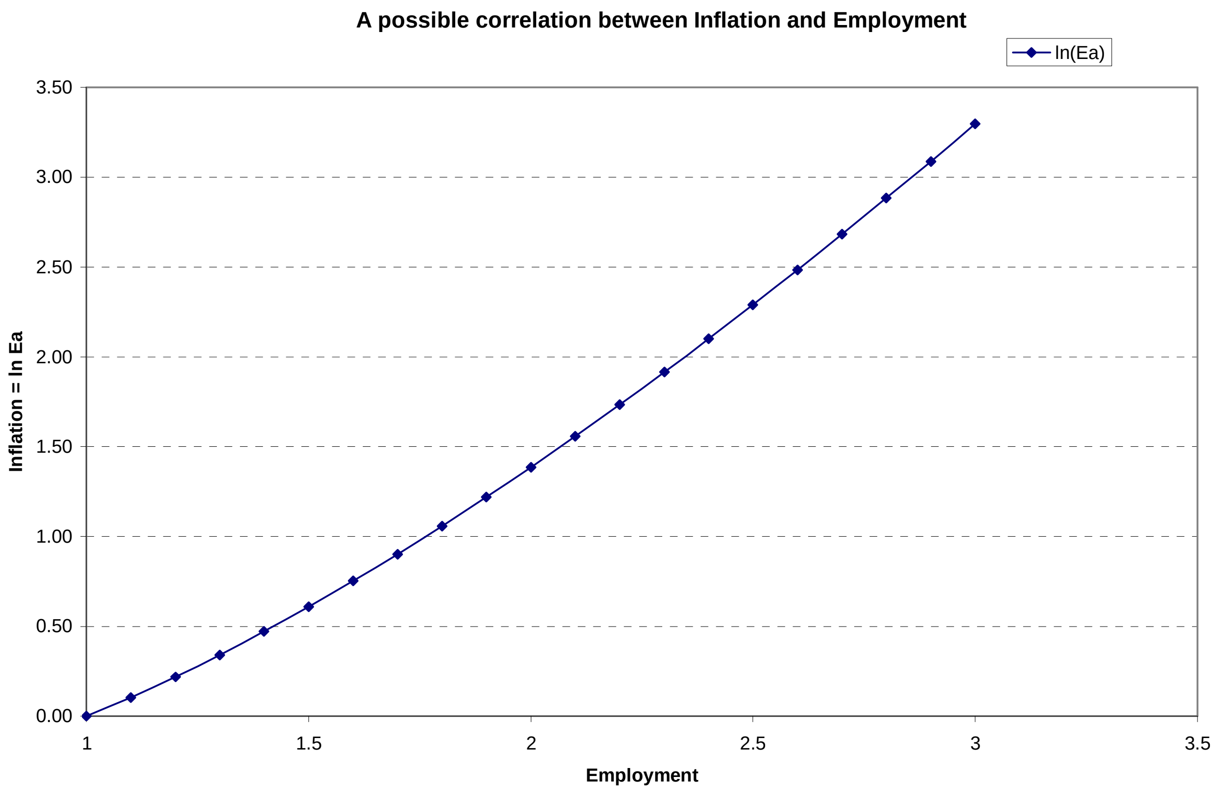
Indices	**compounding**	**growth**	**sub-divide**	**4**	**α**
2	**multiply**	**real terms**	**power**	**population**	**2.718**
nominal	**bacteria**	**e**	**change**	**B**	**argand**

Logarithms - ANSWERS

ACTIVITY ONE

Task One

α	E	Eα	I = ln(E^{α})
2	1	1.00	0.00
2	1.1	1.11	0.10
2	1.2	1.24	0.22
2	1.3	1.41	0.34
2	1.4	1.60	0.47
2	1.5	1.84	0.61
2	1.6	2.12	0.75
2	1.7	2.46	0.90
2	1.8	2.88	1.06
2	1.9	3.39	1.22
2	2	4.00	1.39
2	2.1	4.75	1.56
2	2.2	5.67	1.73
2	2.3	6.79	1.92
2	2.4	8.18	2.10
2	2.5	9.88	2.29
2	2.6	11.99	2.48
2	2.7	14.61	2.68
2	2.8	17.87	2.88
2	2.9	21.93	3.09
2	3	27.00	3.30



Task Two

β= ln0.06/ln2 = -4.05 (to 2dp)

ACTIVITY TWO

Task One

Logarithms are a useful economic tool, which are closely related to powers and **indices**.

We know that 16 = $2^{\mathbf{4}}$ where the number **4** is the **power** or exponent. It is sometimes also known as the index.

Logarithms are particularly useful when analysing rates of **change** and **growth**. A pharmaceutical company, for example, might want to model rates of growth of **bacteria** or an economist might be interested to see how **population** changes over time.

Problems concerning how much interest an investor can expect to receive or how much a sum of money would be worth in **real terms** after a period of inflation are essentially issues surrounding **compounding**. These can be easily solved using logarithms.

In a simple logarithmic expression such as $A=B^{\alpha}$ we can rearrange using logarithms to show that lnA=**αlnB** where ln is the natural logarithm. A natural logarithm simply means a logarithm to the base **e** where e is a constant approximately equal to **2.718**.

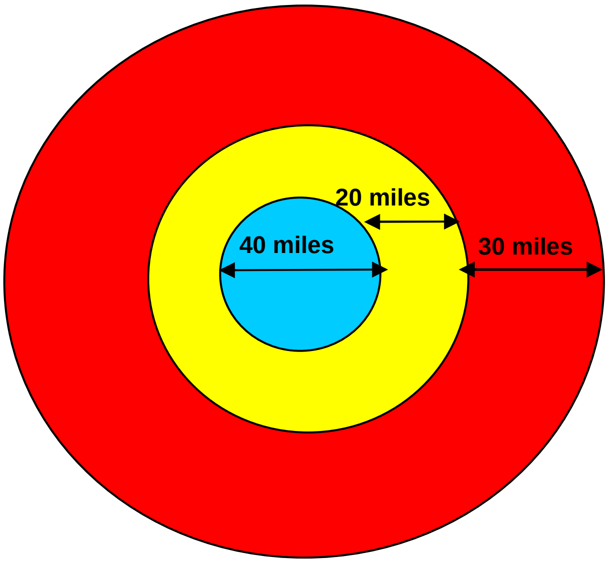
Basic rules of algebra - ACTIVITIES

ACTIVITY ONE

Learning Objectives

LO1: Students to learn how to use simple algebraic formulae

An international space agency is planning to land a robot on Mars. The scientists produce a simple map of Mars and identify 3 zones - red, yellow and blue – upon which the robot could successfully land.



The zones are drawn as concentric circles. The width of the red zone is 30 miles, yellow zone 20 miles and the blue zone has a diameter of 40 miles.

Task One

Calculate the total area of the three zones

Task Two

Calculate the area of the red, blue and yellow zones.

ACTIVITY TWO

Learning Objectives

LO1: Students learn how to independently create a formula

LO2: Students learn how to apply their formula to solve a simple problem

Students can work in pairs to discuss how best to represent Claire's annual income

Task One

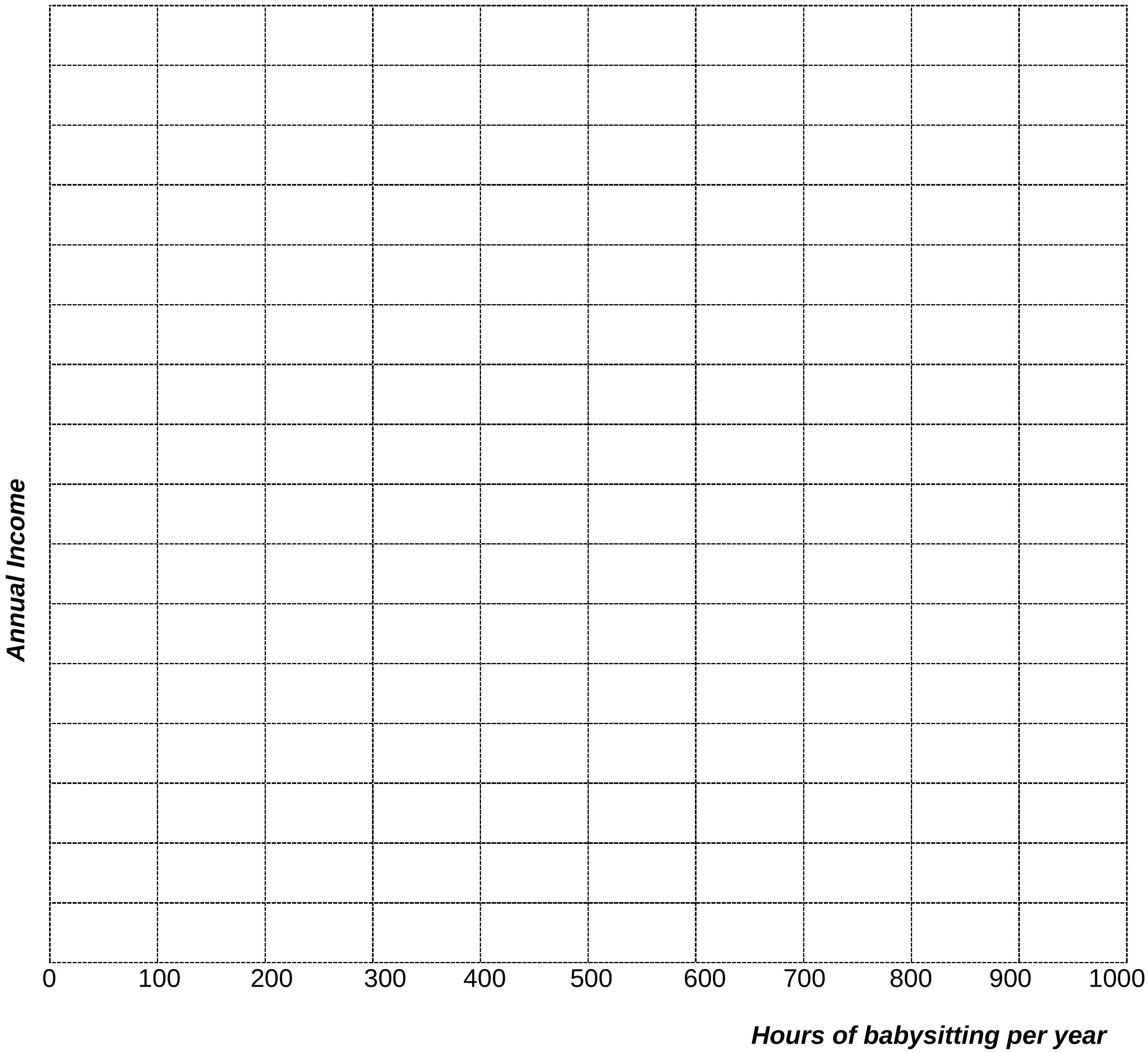
We are told that Claire's income is determined by three separate activities: her salary of £35,000, her part-time babysitting for which she charges £8.00 per hour and the small income she receives from the Government as a grant for her child. The grant is worth £5000 per year.
Create a simple algebraic formula which could be used to calculate her total annual income

Task Two

Graph the income for Claire on the simple graph below. You will need to first:

i) label **both axes**; and

ii) consider a sensible scale for the vertical or y-axis.



Task Three

Claire's total income is taxed at 22%. Write a simple algebraic expression for her post-tax income.

Task Four

(a) Claire is delighted to be told she will receive a **salary rise** of 5% next year. What will her post-tax income formula be now?

(b) If Claire is told she will receive a 5% salary rise every year for *h* years, what will the new formula for her post-tax income be?

Basic rules of algebra - ANSWERS

ACTIVITY ONE

Task One

$A = \prod r^2 = 3.141 \times 70^2$

Area = 15393.8 miles^2

Task Two

Total	blue	yellow	red
15393.80	1256.637	3769.911	10367.26

ACTIVITY TWO

Task One

Let: Y = Claire's total annual income

S = Claire's annual salary

B = Claire's income from babysitting

G = Claire's Government grant

n = the number of hours Claire babysits in a year

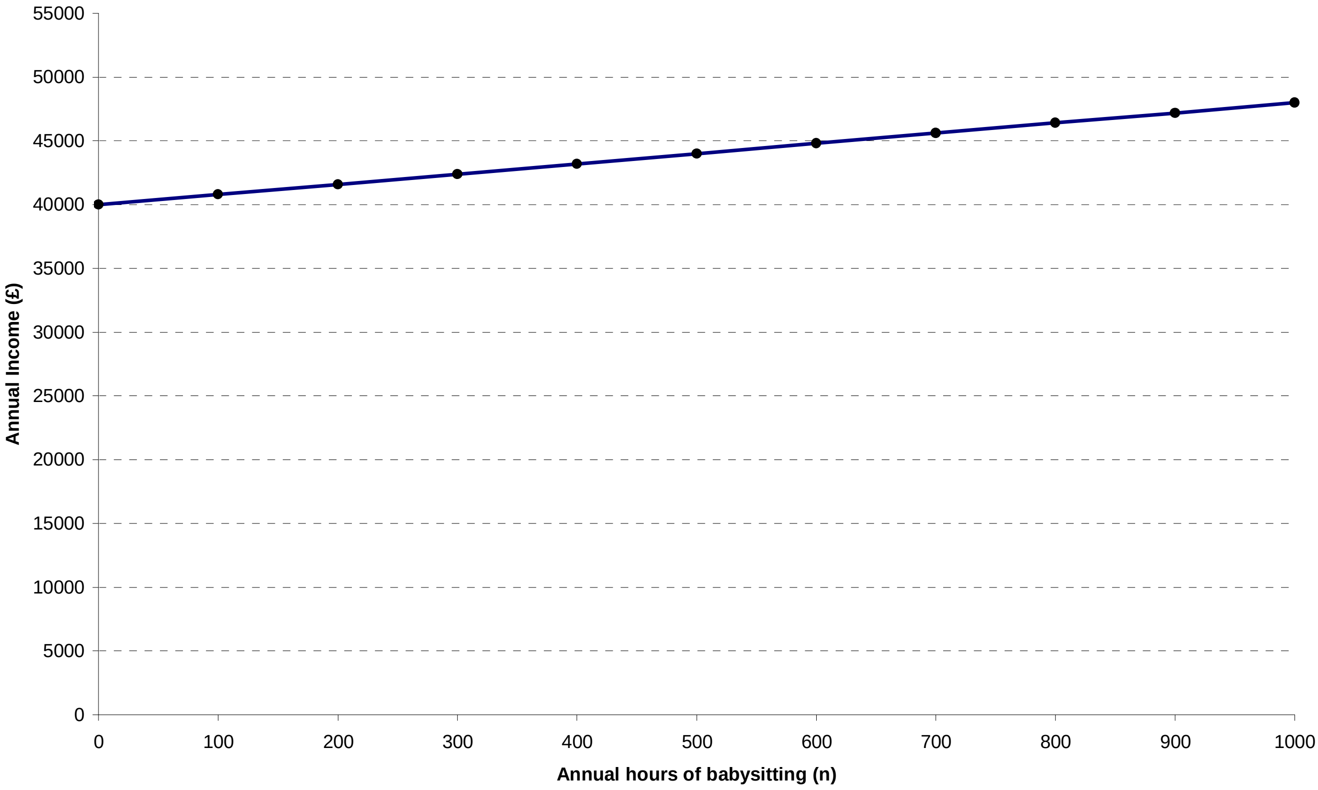
Then: Y = S + B + G

Y = £35000 + 8n + £5000

Y = £40000 + 8n or,

Y = 8(£5000 + n)

Task Two



Task Three

Y = 0.78(40000 + 8n)

Y = 31200 + 6.24n

Task Four

(a) Y = (1.05 x 31200) + 6.24n

Y = 32760 + 6.24n

(b) Y = (31200 x 1.05^{h})+ 6.24n