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Teaching and Learning

Guide 2:

Linear Equations

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Section 1: Introduction to the guide

This Guide focuses on ways to help students to grasp the meaning, significance and practical application of the equation of a straight line. Students will need to have opportunities to achieve a number of learning outcomes including:

- recognising the intercept and slope when presented with such an equation and to be able to confidently construct the line;
- understanding the concept of a Cartesian graph with careful definitions of key terms such as axes and coordinates;
- graphing two-dimensional shapes calculating the area;
- learning the concept of slope and negative gradients and a brief discussion of special cases of zero slope and infinite slope; and
- the concept of the linear equation, using the standard notation $y = a + bx$.

Underpinning this conceptual material is a range of activities to help students apply their knowledge to real world situations. For example, the framework of linear equations is used to produce some Engel curve analysis.

Section 2: Graphs and co-ordinates

1. The concept of graphs and co-ordinates

Students will almost certainly know what is meant by the term, 'graph' but this understanding could be quite simple and descriptive. Lecturers might find students benefit from a simple overview which confirms that:

- A graph consists of an *x*-axis (horizontal axis) and *y*-axis (vertical axis), and a particular point on the graph is identified by its co-ordinates, (*x*, *y*);
- The first co-ordinate is the horizontal position of the point; the second co-ordinate is the vertical position.

Figure 1 below could be used to illustrate the main features of a graph and to demonstrate the plotting of points. The two axes intersect at a point known as the "origin" and labelled "0". Having drawn the axes, the graph contains four regions, or "quadrants", which it is natural to label as: North-East (NE); South-East (SE); South-West (SW); North-West (NW).

Figure 1: Plotting points on a graph
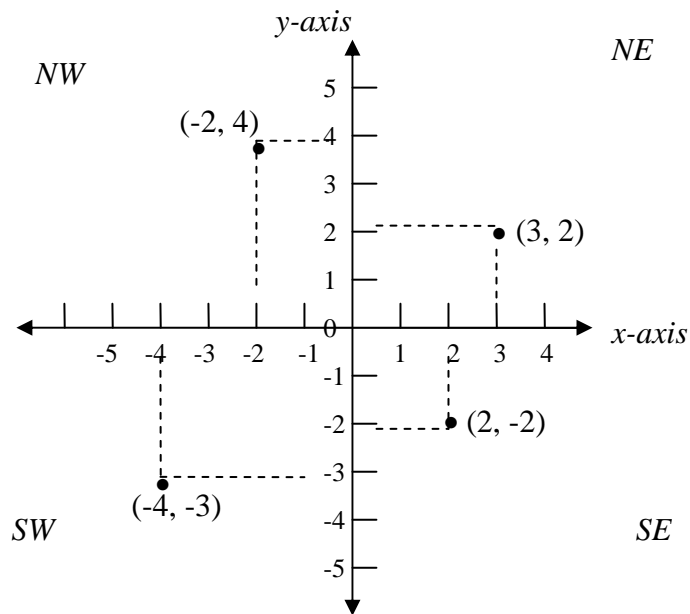
2. Presenting the concept of graphs and co-ordinates

Lecturer might find it useful to go through some simple examples using the quadrant framework in Figure 1. For example, the point (3, 2) is plotted and this point is in the North-East quadrant of the graph. To locate the point, the student might be best advised to start at the origin, move along by 3 units (the first co-ordinate), and then move up by 2 units. Then a large dot, or a small cross, is placed at this point, and labelled with the co-ordinates presented in brackets.

If the first coordinate is negative, the first mover from the origin is to the left. If the second coordinate is negative the second move is downwards. For example, the point appearing in the South-West quadrant, (-4, -3), is reached by moving 4 units to the left, followed by 3 units downwards. This simple practice is likely to pay dividends later on when students will need to independently construct linear equations and graphs.

3. Delivering the concept of graphs and co-ordinates to small or larger groups

Larger groups could be given a Powerpoint presentation which summarised the main features of graphs and co-ordinates but could be encouraged to think of ways in which co-ordinates are used in everyday life e.g. Ordinance Survey maps, Air Traffic Control, Global Positioning Software such as in-car navigation "Tom Tom" systems.

There is considerable scope for small groups to work together on simple problem sets. For example, a whiteboard could be prepared with the four quadrants and small groups could then take it in turns to test each other on their knowledge of co-ordinates such as plotting points. This could be extended with small groups or teams writing a "Tue or False" handout which would set out a graph with say 10 plotted and labelled points. Each group would then write 12 statements relating to the graph and give to the other groups(s) to identify which assertions were correct and which were false. An example is given below:

Plotting and Understanding Co-ordinates
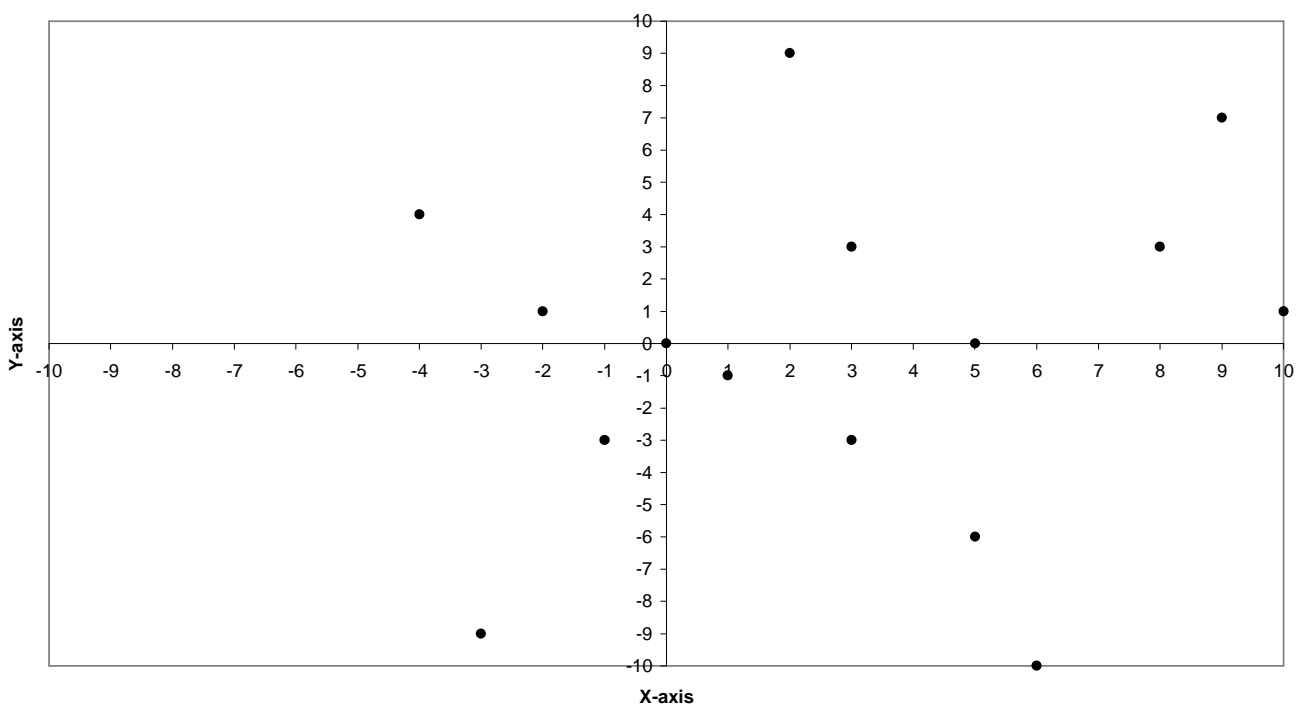
Assertion 1: There are no points which have both negative x-axis and negative y-axis values. (which is clearly false)

Links with the online question bank

The questions for the Guide can be found at

http://www.metalproject.co.uk/METAL/Resources/Question_bank/Calculus/index.html

These cover a wide range of questions relating to co-ordinate geometry.

Video clips

The video clips for this Guide can be found at:

http://www.metalproject.co.uk/Resources/Films/Linear_equations/index.html.

Students might find it helpful to watch the first clip "2.01 Linear Fractions - Budget Lines" which provides a straightforward overview of axes and points and also a practical application. If this clip is used as an introduction, colleagues might wish to stop the film at around 4.00 minutes (after this point the clip looks at an algebraic expression of the budget problem which could be inappropriate for an introduction).

4. Discussion Questions

As mentioned above, students could be encouraged to reflect on practical ways which co-ordinates are used in business and industry. This could feed into an interesting discussion on how important this concept is. A mind map or spider diagram could be created in a student tutorial or workshop to summarise the discussion.

5. Activities
ACTIVITY ONE
Learning Objectives

LO1. Students to consolidate meaning of graphs and co-ordinates

LO2. Students to practice plotting simple points

Task One

Draw a graph with each axis having range -5 to +5. The following sets of coordinates, when connected by straight lines in the order specified, each trace out one of the following objects: a square; a rectangle; a diamond; a triangle. Which set of coordinates traces out which object?

A: $(-2,1) \rightarrow (0,-1) \rightarrow (2,1) \rightarrow (0,3) \rightarrow (-2,1)$

B: $(3,0) \to (4,2) \to (3,4) \to (2,2) \to (3,0)$

C: $(0,-2) \to (3,1) \to (4,-2) \to (2,-2) \to (0,-2)$

D: $(1,-2) \to (-2,4) \to (0,5) \to (3,-1) \to (1,-2)$

ACTIVITY TWO

LO1. Students to understand the practical significance of choosing which axis to use for plotting variables

Consider the following pairs of variables. If you were going to plot one against the other, on which axis would you place them? First of all decide (if possible) which variable is independent, and which dependent.

(i) Annual Income of an individual and total Annual Expenditure by the individual.

(ii) Weekly income of an individual and number of hours worked per week by the individual.

(iii) Annual Income of an individual and number of years of education of the individual.

(iv) Price of fuel and quantity consumed of fuel by UK households.

(v) Price of fuel and quantity of fuel supplied to world market by oil-producing countries.

(vi) Number of students attending UK Universities and average tuition fees

<u>ANSWERS</u>

ACTIVITY ONE

A: Square (and illustrated below in **Figure 2**)

B: Diamond

C: Triangle

D: Rectangle

Figure 2
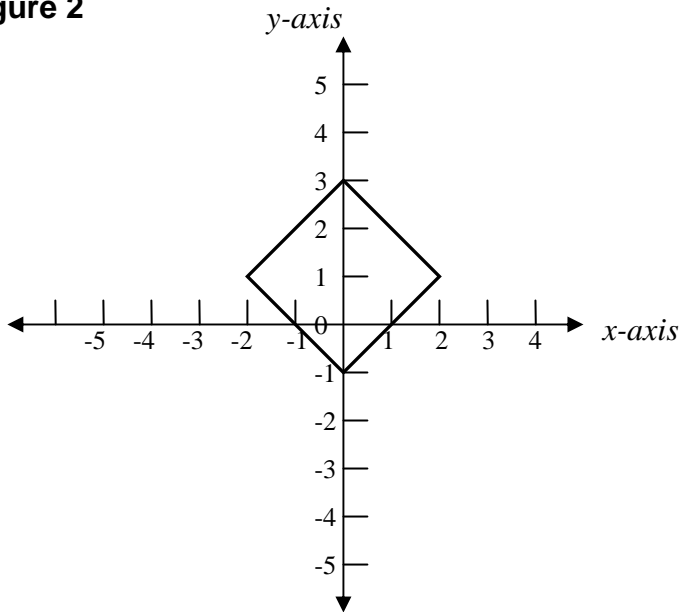
ACTIVITY TWO

(i) Annual Income of an individual **(x)** and total Annual Expenditure by the individual **(y)**

(ii) Weekly income of an individual **(y**) and number of hours worked per week by the individual **(x)**

(iii) Annual Income of an individual **(y)** and number of years of education of the individual **(x).**

(iv) Price of fuel **(y)** and quantity consumed of fuel by UK households **(x).**

(v) Price of fuel and **(y)** quantity of fuel supplied to world market by oil-producing countries **(x)**

(vi) Number of students attending UK Universities **(y)** and average tuition fees **(x)**

6. Top Tips

Remind students that when plotting a point on a graph they should start at the origin, move along by the amount of the first co-ordinate (to the right if positive; to the left if negative), then move up (or down if negative) by the amount of the second coordinate. Kinaesthetic learners could practice this simple tip by tracing this out with their finger on a whiteboard when plotting points.

7. Conclusion

This first part of linear equations is simple and straightforward but crucial to students being able to take the next steps towards more advanced material. Students will probably benefit from lots of practice drawing graphs and plotting points correctly. It is important that students are reminded to label **both** axes and to independently title graphs: too often we see unlabelled graphs without titles!

Section 3: Finding the area of a region of a graph

1. The concept of areas of a region of a graph

Finding the area of a region of a graph is an important skill when it comes to evaluating a number of economic problems or issues e.g. consumer surplus, producer surplus, economic rent and transfer earnings. Students could be encouraged to research one or more of these concepts so they can see in parallel the practical significance of the mathematics they are learning.

2. Presenting the concept of areas of a region of a graph

Students need know that:

-   The area of a square is calculated by simply squaring one of the sides;
-   The area of a rectangle is calculated by multiplying the base by the height; and
-   The area of a triangle can be measure by multiplying half of the base by the height.

Students could be a given a simple formula sheet with simple illustrations to which they can later refer. Alternatively, lecturers might to create a more active learning activity by asking students to draft their own formula sheet. The clearest formula sheet could be used as an exemplar and posted on an intranet.

3. Delivering the concept of areas of a region of a graph to small or larger groups

It is likely to be helpful if lecturers could provide some simple example perhaps in prepared handout or in a presentation to confirm exactly what is meant by a region or area: both of these

terms are likely to be unclear to students. This approach could be sued both with small and larger groups.

If students have researched one of the practical applications (see 1. above) then this could be to illustrate the concept e.g. a simple supply and demand diagram could readily be extended to show consumer and producer surplus and so the ideas of regions or areas of a graph.

As with graphs and co-ordinates, students will benefit from having plenty of practice of plotting points and independently constructing areas in a graph. Some examples of how this might be done are given below. Colleagues might find it easier to support groups which are smaller with linking the activities of plotting areas to practical economic applications such as consumer and producer surplus.

Links with the online question bank
The questions for the Guide can be found at
http://www.metalproject.co.uk/METAL/Resources/Question_bank/Calculus/index.html
These cover a wide range of questions relating to co-ordinate geometry. Links could be made to complementary topics such as supply and demand and questions on this can be found at:
http://www.metalproject.co.uk/METAL/Resources/Question_bank/Economics%20applications/index.html

Video clips
The first Flash animation in Series Two (Linear Fractions – Budget Lines) at:
http://www.metalproject.co.uk/METAL/Resources/Films/Linear_equations/index.html
offers an effective overview of the consumer budget line and this can easily be developed to show areas or regions on a graph, i.e. all of the points to the left of the budget line are feasible as well as all of the points along the budget line.

4. Discussion Questions
There is a lot of scope for students to share and discuss the practical significance of regions and areas of a graph. If students are asked at the beginning of this material to research topics such as producer and consumer surplus this could be revisited in a discussion group or workshop. Higher ability groups could look at higher level materials such as international trade

and identifying areas on graph to show the size of imports and exports into an economy. Lecturers might want to create a resource which requires students to plot a simple supply and demand diagram in a four quadrant graph and then to use this as a platform for a wider discussion on measuring surpluses.

5. Activities
ACTIVITY ONE

LO1: Students to independently construct regions of a graph

LO2: Students to independently calculate the area of a region of a graph

TASK ONE

(a) Plot the following sets of coordinates:

$$(-2,1) \rightarrow (0,-1) \rightarrow (2,1) \rightarrow (0,3) \rightarrow (-2,1)$$

(b) Confirm that when these points are connected, a perfect square is traced out.

(c) You will see that since the sides of the square are not parallel with the axes, it is not immediately obvious what the length of a side is. One way to compute the area of this square is to consider the larger square shown in Figure 3, given by:

$$(-2,-1) \rightarrow (2,-1) \rightarrow (2,3) \rightarrow (-2,3) \rightarrow (-2,-1)$$

Figure 3
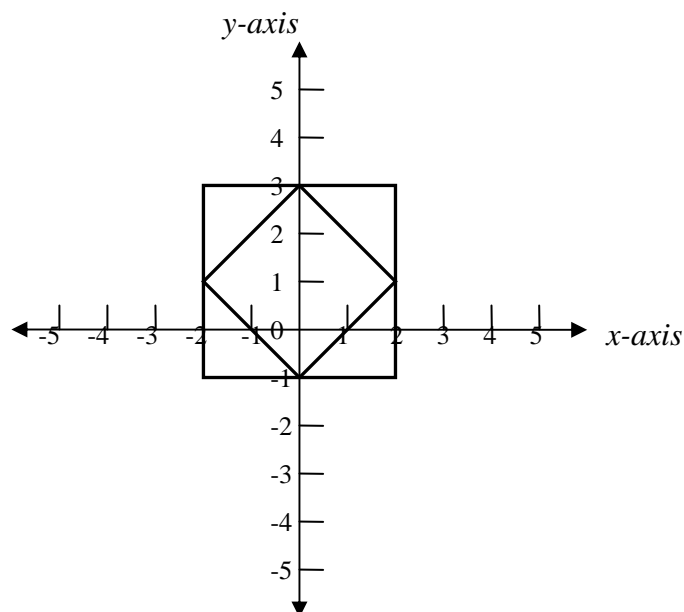
This larger square has sides of length 4 and therefore has area 16 (i.e. 4^2). The area of the smaller square can be deduced by subtracting from 16 the areas of the four triangles outside the smaller square but inside the larger:

Confirm that the area of the smaller square can be calculated by:

$$Area = 16 - \left(4 \times \frac{1}{2} \times 2 \times 2\right) = 16 - 8 = \underline{\underline{8}}$$

(d) Plot these shapes (from Section 2 above) and calculate their areas:

B: $(3,0) \rightarrow (4,2) \rightarrow (3,4) \rightarrow (2,2) \rightarrow (3,0)$

C: $(0,-4) \rightarrow (2,-2) \rightarrow (4,-4) \rightarrow (2,-6) \rightarrow (0,-4)$

D: $(4,0) \rightarrow (4.5,1) \rightarrow (5,0) \rightarrow (4.5,-1.0) \rightarrow (4,0)$

ANSWERS

Area of A =8

Area of B = 4

Area of C =8

Area of D = 1

6. Top Tips

Give students opportunities to undertake peer assessment e.g. students creating more complex areas or regions and giving co-ordinates to other students or pairs. This could be developed into a separate exercise on estimate on: how easy is it to estimate the area on a graph without the use of measurements or formulae?

7. Conclusion

This topic is an important links between graphical and algebraic relationships. Students will benefit from having time to practise plotting and measuring areas. There are many obvious links into micro- and macroeconomics from this point including consumer and producer

surplus. Colleagues might wish to use this topic as a springboard into more detailed and theoretical topics such as indifference curve analysis.

Section 4: Slope (or gradient) of a line on a graph

1. The concept of the slope

Students will need to be given a simple idea of what is meant by a slope. For example, referring to road signs, which present the steepness of a hill as "one-in-ten" or "one-in-six". Students will need to understand these are the same as the gradient of the hill and to understand that a "one-in-six" hill is means that we rise one metre for every six metres we move forward. A simple illustration such as Figure 4 might be useful:

Figure 4: Illustrating Gradients

1 metre

6 metres
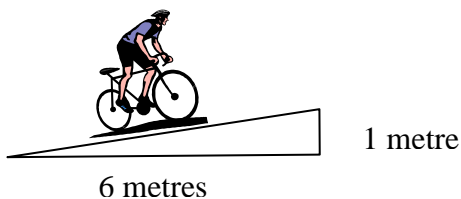
2. Presenting the concepts of slopes and gradients

Students will find it easier to relate simple diagrams such as Figure 4. Lecturers might want to give students a range of gradients, both numerically and through graphical illustration. For example, the steepest street in the world is in Dunedin, New Zealand with a gradient of one in 2.66! This is shown by Figure 5 below:

Figure 5: The Steepest Street in the World

1 metre

2.66 metres
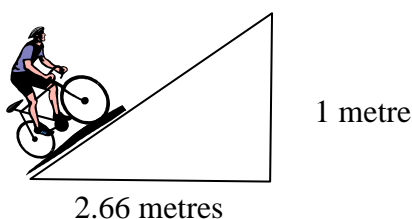
Students will need to understand that slopes are usually represented with a single number. An that this number is the vertical distance divided by the horizontal distance. The example of Dunedin could be used as an example: the slope of the street in Dunedin is 1÷2.66=0.38.

Students will need to understand that the slope of the line connecting the two points on a graph (x_1,y_1) and (x_2,y_2) is given by:

$$\frac{y_2 - y_1}{x_2 - x_1}$$

and that $y_2 - y_1$ is the vertical difference between the two points, and $x_2 - x_1$ is the horizontal difference. This could be linked back to earlier work on co-ordinates.

3. Delivering the concept of a gradient of a straight line to smaller or larger groups

This topic can be readily differentiated. Students with little mathematical experience might respond well to graphical explanations with plenty of scope for students to practise simple calculations of gradients. Students can be given A3 sheets which they turn into a grid and in pairs select points and then work out gradients.

Higher ability students could move quickly onto economic applications of gradients and the natural links that exist with elasticities. Students could create their own problem sets based on supply and demand. For example, students could work in pairs graphing a supply and demand curve (and could make this is as realistic as possible by using a "real product" with a typical market equilibrium price) and then drafting questions and answers for peers to work through. In this way, students create their own worksheets, have chances to undertake peer assessment and are engaged in (more) active learning.

Links to the online question bank

The questions for the Guide can be found at

http://www.metalproject.co.uk/METAL/Resources/Question_bank/Calculus/index.html

These cover a wide range of questions relating to co-ordinate geometry. Links could be made to complementary topics such as supply and demand and questions on this can be found at:

http://www.metalproject.co.uk/METAL/Resources/Question_bank/Economics%20applications/index.html

Video clips

The video clips for this Guide can be found at:

http://www.metalproject.co.uk/Resources/Films/Linear_equations/index.html. The clips 2.02 –
2.04 (inclusive) would be an effective way to show how simple straight line equations have a
practical significance. Students might want to think of any other examples where a linear
relationship might exist e.g. costs and benefits, consumption and saving?

4. Discussion

The paired work set out in 3 above could form the basis for a plenary workshop and students
could feedback they work they did on supply and demand. Discussion questions could cover
the link between the steepness of the demand/supply curve and price elasticity. The
discussion could also cover which products might have steep demand curves and which would
tend to have more shallow gradients and what we can infer about the nature of these proucts
given this data.

5. Activities
LO1: Students to learn the formula for calculating a gradient of a straight line
LO2: Students to learn how to calculate the gradient of a straight line

TASK ONE

State the formula for measuring the gradient of a straight line

TASK TWO

   (a) Graph the following two points on a graph: (1,2); (2,4).

   (b) What is the slope of this straight line?

   (c) Interpret the meaning and significance of the "+" sign in front of the gradient figure.

TASK THREE

Choose one of the following pairs of points. For the pair of points you have chosen, carry out
the following tasks:

(1)   Plot the two points on a graph.(2) Draw a straight line passing through the two points

(3)   Find the slope of the line; is the line upward-sloping, downward-sloping, or neither?

(4)   Report the coordinates of a different point on the same straight line

Pairs of points:

A: (-2, 4) and (2, 0)

B: (-3, -1) and (4, -1)
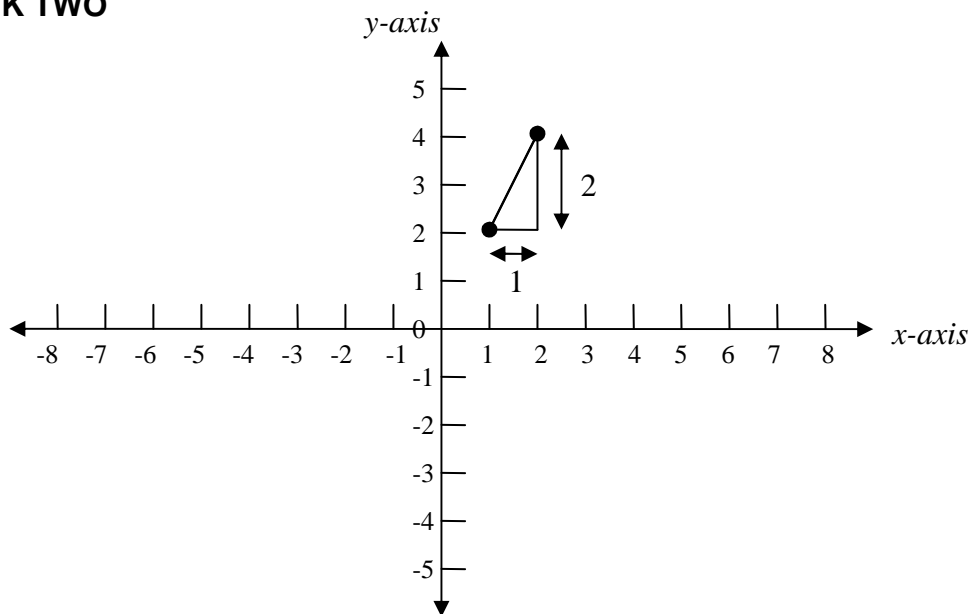
C: (1, 4) and (1, -3)

D: (-1, -2) and (5, 1)

ANSWERS

TASK ONE

$$\frac{y_2 - y_1}{x_2 - x_1}$$

TASK TWO

(a)

(b) $slope = \dfrac{4-2}{2-1} = +2$

(c) The positive sign means that the gradient is upward sloping from left to right. If the sign had been negative this would have indicated that the line was downward sloping from left to right.

TASK THREE

Answers depend on the points chosen.

6. Top Tips

Students need to appreciate that whenever the slope of a line is positive the line is upward-sloping and that a negative slope implies a "downward-sloping" line, that is, one that falls as we move from left to right. Note should be made of the two important special cases:

- When the slope of a line is zero, the line is neither upward-sloping not downward-sloping, it is <u>horizontal; and;</u>
- When the slope of a line is infinity (∞), the line is <u>vertical</u>.

7. Conclusion

This material is particularly important because it has obvious linkages into differentiation and rates of change. Students who have a sound grasp of gradients and changes will have a good foundation for learning differentiation and integration in later guides.

Section 5: The equation of a straight line: intercept and slope

1. The concept of the equation of a straight line

Most students will recognise the term "equation" but surprisingly few have a solid understanding of exactly what it means. It would be useful to start with a simple re-statement and example (see below).

Example

Definition of an equation: an equation is any expression containing an "equals" sign.

Consider the following equation involving the two quantities, x and y:

$$y = -3 + 2x$$

The equation can easily be used to work out the value of *y* for any given value of *x*. All you need to do is insert your given value for *x*, and then evaluate the expression on the right hand side of the equation – this will be the corresponding value of *y*.

Repeating this for other values of *x* we get:

x	y
-2	-3 + 2(-2) = -7
-1	-3 + 2(-1) = -5
0	-3 + 2(0) = -3
1	-3 + 2(1) = -1
2	-3 + 2(2) = 1
3	-3 + 2(3) = 3
4	-3 + 2(4) = 5

2. Presenting the concept of the equation of a straight line

If lecturers have adopted the example above, they might find it useful to supplement this with the graphical representation shown below.

That is, to plot *y* against *x* on a graph as illustrated in Figure 6 by plotting the points given by the pairs of co-ordinates (e.g. (-2, -7) and (4,5)) in the table above:

Figure 6: Drawing a straight line on a graph
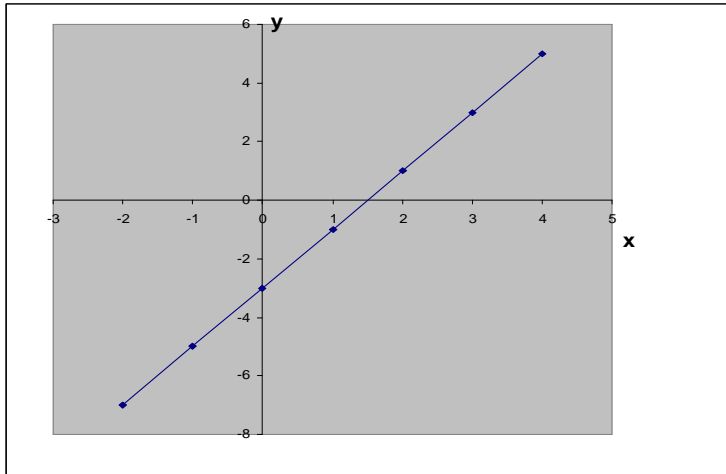
Students will need to be aware that when we say we are plotting _y against x_, we imply that _x_ is measured on the horizontal axis, and _y_ on the vertical. Colleagues might want to ask students to verify that if we plotted _x_ against _y_, it would be _y_ on the horizontal and _x_ on the vertical.

From this students can see that the line which shows the relationship between _x_ and _y_ is a straight line and that the straight line crosses the vertical axis at -3 (the <u>intercept</u> of the straight line.)

Similarly, this graph can be used to show when _x_ increases by one, _y_ always increases by 2. i.e. the slope is +2

3. Delivering the concept of the equation of a straight line to smaller and larger groups

The graphical overview could be a neat way to introduce the idea of a straight line and how the linear relationship between _x_ and _y_ can be calculated. Groups will need a more formal expression too, namely:
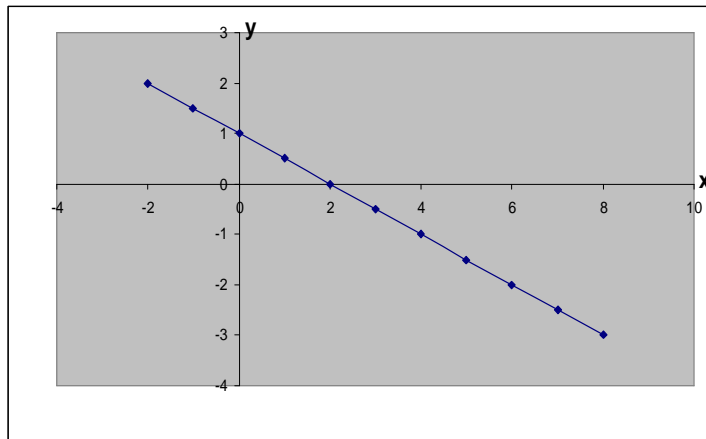
$$y = a + bx$$

The intercept is _a_ and the slope is _b._ Another example, if the equation is:

$$y = 1 - \frac{1}{2}x$$

Here the intercept is 1, and the slope is $-\frac{1}{2}$, illustrated in Figure 7.

Figure 7: A downward-sloping straight line

Students will want to note that the negative slope implies that there is a negative relationship (i.e. as one increases the other falls) between the two variables *x* and *y*.

Students could be asked to think of their own straight line equations and to graph these. They could also do this in reverse by graphing a straight line and then by inspecting the drawn line, arrive at accurate figures for the slope and intercept. Lecturers could project graphs onto a whiteboard with simple multiple choice questions. This might be particularly useful for large groups.

Students might find it useful to look through the worked examples in Section 6 at the end of this guide.

Links with online question bank

The questions for the Guide can be found at

http://www.metalproject.co.uk/METAL/Resources/Question_bank/Calculus/index.html

These cover a wide range of questions relating to co-ordinate geometry. Links could be made to complementary topics such as supply and demand and questions on this can be found at:

http://www.metalproject.co.uk/METAL/Resources/Question_bank/Economics%20applications/index.html

Video clips

Students might benefit from seeing the video clip which introduces the next 'stepping stone':
linear equations. This can be found at :

http://www.metalproject.co.uk/Resources/Films/Linear_equations/index.html as clip "2.05
Linear Equations in Goods and Labour Markets"

4. Discussion

This material could easily be developed to include a wider discussion on absolute and
proportional changes using straight line equations. Students could consider for example if *x*
rises by 1%, by what percentage does *y* rise as a result? Similarly students could examine that
if quantity changes from *a* to *b*, the <u>absolute</u> change in the quantity is *b-a* whilst the

proportional change is $\dfrac{b-a}{a}$, and the percentage change is $100\times\dfrac{b-a}{a}$. There are obvious links

to fractions and percentages too.

5. Activities

TASK ONE

LO1: Students to learn to interpret straight line equations

**LO2: Students to apply their knowledge and understanding of a straight-line equation to
simple Engel curves.**

*Note: Students might find it useful to review the worked examples and background material
provided in Section 6.*

The following equations each show the relationship between household income per week (*x*)
and household expenditure on a particular good per week (*y*):

(1) $y = 100 + 0.3x$

(2) $y = 200 - 0.15x$

(3) $y = 25$

(4) $y = -200 + 0.05x$

Answer the following questions relating to equations (1)-(4).

(a)   What are these equations called by Economists?

(b)   To what sort of good do you think that each equation relates? Explain your answers.

(c)   In each case, calculate weekly expenditure on the good for a household earning 1000 per week.

(d)   In each case, where applicable, find the income level at which expenditure equals zero.

TASK TWO

There are two Pizzerias in your street. *Mama Mia's* charges £6 for the base and £1 for each topping. *Pepe's Pizzas* charges £3 for the base and £2 for each topping. You have heard that they both supply pizzas of similar size and quality. Let *x* be the number toppings you want and let *y* be the amount you pay for the pizza.

(a) Draw a graph with a horizontal scale of 0 to 6 and construct a line for each Pizzeria showing price against number of toppings. Label the two lines.

(b) Using the graph, consider how the number of toppings would determine your choice of Pizzeria. Is there any situation in which you would be indifferent?

TASK THREE

LO1: Students to learn how to independently write simple straight line equations

LO2: Students to learn how make inferences using a straight line.

LO3: Students to learn how to invert a straight line equation.

It is well known that to convert temperature in degrees Centigrade to temperature in degrees Fahrenheit, you do the following, in the specified order:

Multiply by 9; divide by 5; add 32.

Let *x* be temperature in degrees Centigrade, and let *y* be temperature in degrees Fahrenheit.

(a) According to the above rule, write down an equation which shows *y* as a function of *x*. Your equation should be the equation of a straight line. What is the intercept, and what is the slope?

(b) Sketch the line on a graph, with *x* measured on the horizontal axis, and *y* on the vertical.

(c) Use your equation to answer the following:

    (i) Water freezes at zero degrees Centigrade. At what temperature does water freeze in degrees Fahrenheit?

    (ii) Water boils at 100 degrees Centigrade. At what temperature does water boil in degrees Fahrenheit?

    (iii) "Absolute Zero" is the lowest temperature that it is theoretically possible to attain. It is -273 degrees Centigrade. What is Absolute Zero in degrees Fahrenheit?

(d)

    (i) Next, <u>invert</u> your equation, so that it shows *x* as a function of *y*.

    (ii) Sketch the straight line represented by the new equation on a graph. Note that for this graph, *y* should be measured on the horizontal, and *x* on the vertical.

    (iii) Use this second equation to answer the following. "Normal" body temperature is 98.6 degrees Fahrenheit. What is normal body temperature in degrees Centigrade?

ANSWERS

TASK ONE

(a) These equations represent Engel curves: relationships between the quantity demanded of a good or service and levels of income.

(b) Goods (1) and (4) are normal goods. Good (2) is an inferior good. Good (3) posits there is no relationship between the quantity demanded and income level.

(c) Good 1: $y = 400$, Good 2: $y = 50$, Good 3: $y = 25$, Good 4: $y = -150$

(d) Good 1: x= -£333.33, Good 2: x= £1333.33, Good 3: x =£0, Good 4: x=£4000

TASK TWO

    (a) see below
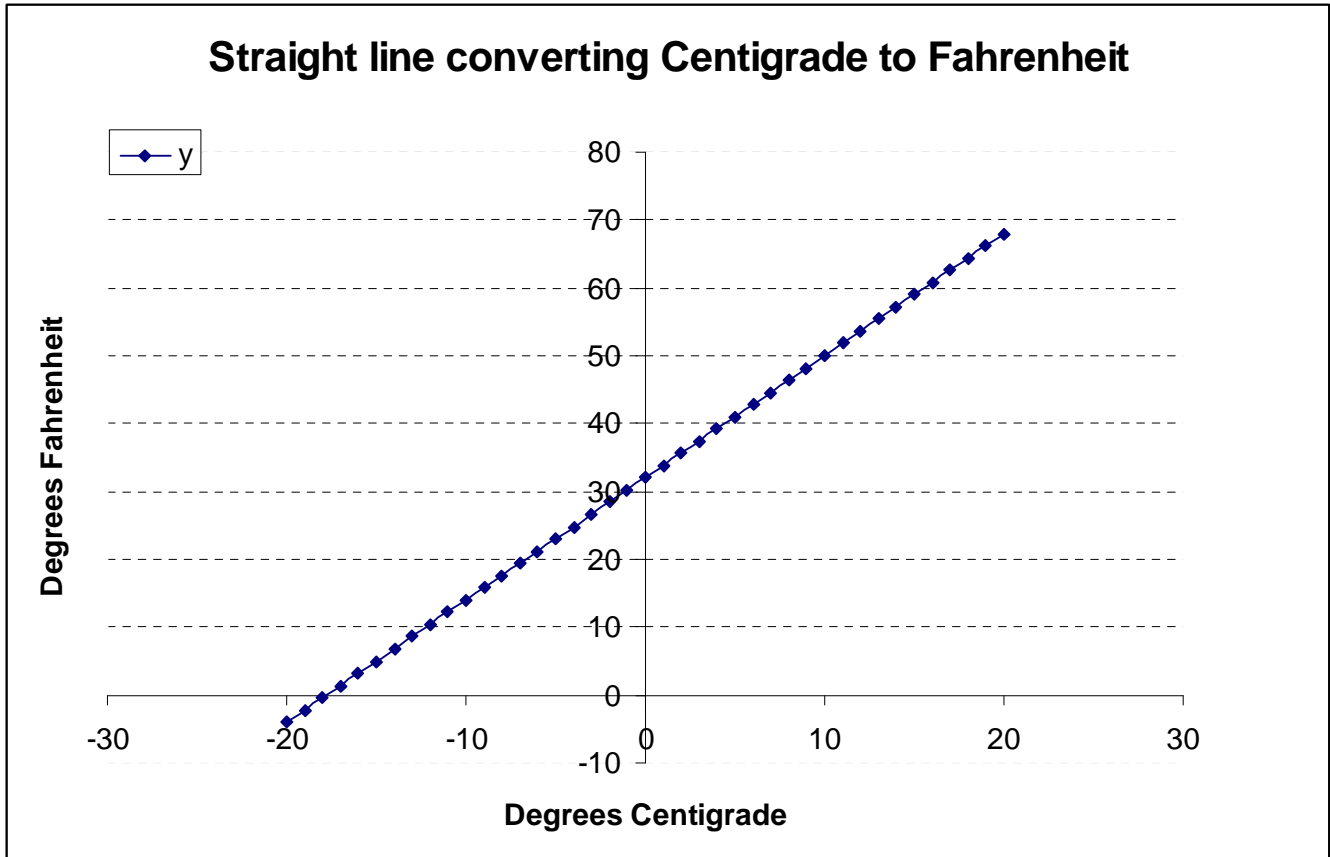
Equations to show the costs of pizzas
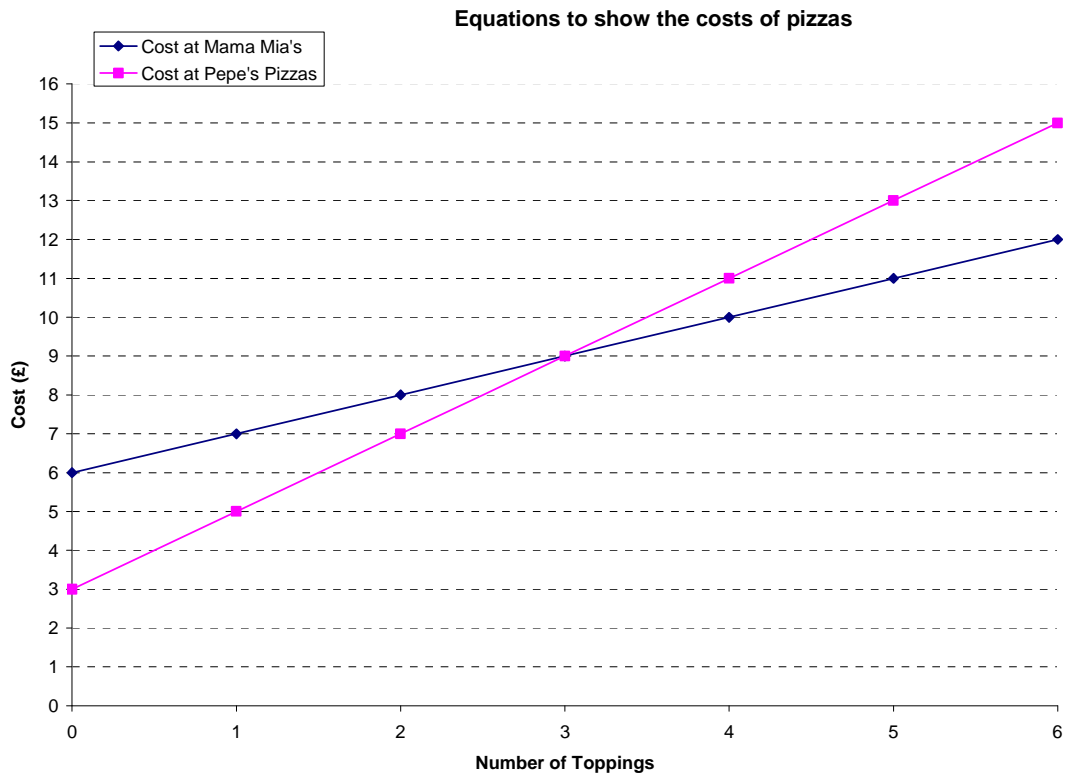
(b) At 3 toppings the cost at either pizzeria is £9 and assuming that quality is identical the consumer would be indifferent.

TASK THREE

(a) $y = 32 + \dfrac{9}{5}x$

(b)

Straight line converting Centigrade to Fahrenheit

(c)

(i) 32^0F

(ii) 212^0F

(iii) -459.4^0F

(d)

(i) The equation for converting temperatures from degrees Centigrade (*x*) to degrees Fahrenheit (*y*) is:

$$y = 32 + \frac{9}{5}x$$

To invert this equation, we do the following:

$$-32 + y = \frac{9}{5}x$$

$$\therefore \frac{9}{5}x = -32 + y$$

$$\therefore x = -32\left(\frac{5}{9}\right) + \left(\frac{5}{9}\right)y$$

$$\therefore \underline{x = -17.778 + 0.556\,y}$$

This equation can be used to convert from Fahrenheit (*y*) to Centigrade (*x*).

ii) and (iii) Using the new graph and/or equation, we get 37^{0}C.

6. Top Tips

Students could be reminded that although we can construct a table of x-values and corresponding *y*-values, with a straight line note that only t<u>wo</u> points are necessary to draw the straight line, preferably two points a reasonable difference apart.

7. Conclusion

This final section pulls together the earlier material on graphing points, areas and regions, slopes and also the equation of the straight line. It is important to ensure this material is contextualised wherever possible and the questions attached at the end of this guide are intended to provide this practical application. They should also help students to prepare for the next section: Linear Equations.

## Section 6: The Equation of a Straight Line: Economic Examples

a) A worked example to understand prices and costs

When entering a taxi in the hours of darkness, you notice that the amount on the meter is pre-set to £4.00. Subsequently the amount on the meter rises by £1.50 for each mile you travel. If you know how far your journey will be can you calculate the total charge before you reach your destination? Let x be the length of your journey in miles, and let y be the total taxi fare. The equation giving total fare for any distance x travelled is:

$$y = 4.0 + 1.5x$$

Again this is recognisable as the equation of a straight line with intercept 4.0 and slope 1.5. At this stage it is worth stressing a point that students often fail to grasp. This point is that when students are asked to interpret the slope in this equation, they often say things like "£1.50 is the price per mile travelled", which is of course incorrect. The error being made here is that a *marginal* concept is being misinterpreted as an *average* concept. A correct interpretation of the slope is that "£1.50 is the amount by which the fare rises if the distance increases by one mile". Of course, average concepts are sometimes of interest, which may need to be more clearly defined as shown in Figure 8. In the graph there are two rays it is the slope of the ray passing through the origin that is imagined, this provides the total average cost per mile of £2.50, unlike the second ray that represents the equation.

Figure 8: marginal and average cost of a mile travelled by taxi
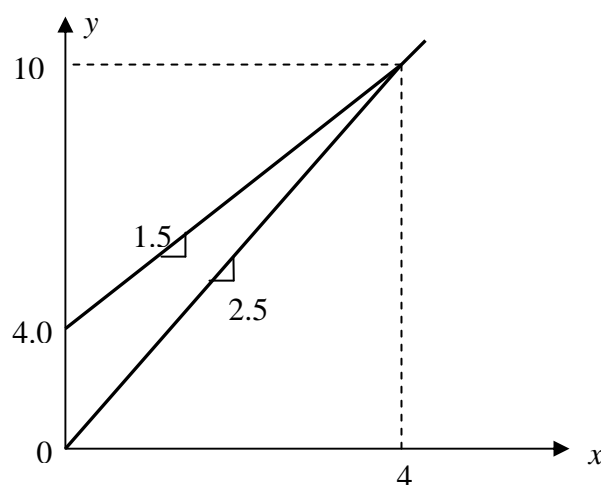
The slope of a line represents the <u>marginal</u> effect of a change in the *x*-variable on the value of the *y*-variable.

It is also useful to introduce the concept here of fixed and variables costs associated with the equation: the intercept represents the fixed cost element of the price and the slope the variable cost. To re-enforce these ideas, examples of naturally occurring situations in which the equation of a straight line is relevant in describing the relationship between two quantities form the following Learning Activity:

Each student should be assigned one of the following four tasks, and having completed it should be prepared to present their answer to rest of the group.

b) Problem sets

Problem 1

You are planning to rent a car for a day. *Hot Wheels* charges a flat fee of £80 for the day. For an identical car, *Top Gear* charges a fixed fee of £50, plus £0.15 for each mile travelled. Let *x* be the number of miles you travel, and let *y* be the total rental charge. Draw a graph with two lines, one for each company, showing total charge against number of miles travelled, using a horizontal scale of 0 to 400. Label the two lines. Use the graph to determine the circumstances under which you would hire from *Hot Wheels*, and those under which you would hire from *Top Gear*. Is there any situation in which you would be indifferent?

Problem 2

Your Students' Union is planning the Graduation Ball. The organisers are trying to decide between two ticket pricing schemes. Under scheme 1, tickets are £30 each, with free unlimited drinks until late. Under scheme 2, tickets are £15 each, and all drinks will be charged at £3 each. Like most students, you are on a tight budget, but you want to go to the ball and maximise your enjoyment. Let *x* be the number of drinks you consume, and let *y* be your total expenditure for the night. Draw a graph with two lines (one for each pricing scheme) showing total expenditure against the number of drinks purchased. Label the two lines. Use the graph to determine the circumstances under which you prefer scheme 1,

and those under which you prefer scheme 2. Is there any situation in which you would be indifferent?

Problem 3

Before taking a trip to Ireland, you visit the bank to get your spending money. You need to convert pounds (£) into euros (€). You notice that the bank charges a fixed fee of £3 (commission), and an additional £0.70 for each euro. Let *x* be the number of euros that you purchase, and let *y* be the total cost of these in pounds. Draw a graph with a line showing total cost in pounds against the number of euros purchased. If you only buy one euro, what is the <u>average</u> cost per euro? What about €100? €1million? What happens to the average cost per euro as the size of the purchase rises?

b) Further worked examples : Engel Curves

In this Section we apply the framework of linear equations to a particular area of microeconomic theory: Engel curve analysis. Let *x* be a household's income, in pounds per week, and let *y* be the household's expenditure on food, again in pounds per week.

The equation connecting *x* and *y* might be:

$$y = 100 + 0.25x$$

This is a straight line with intercept 100 and slope 0.25. We sketch this equation in Figure 9.

Figure 9: Engel curve for food
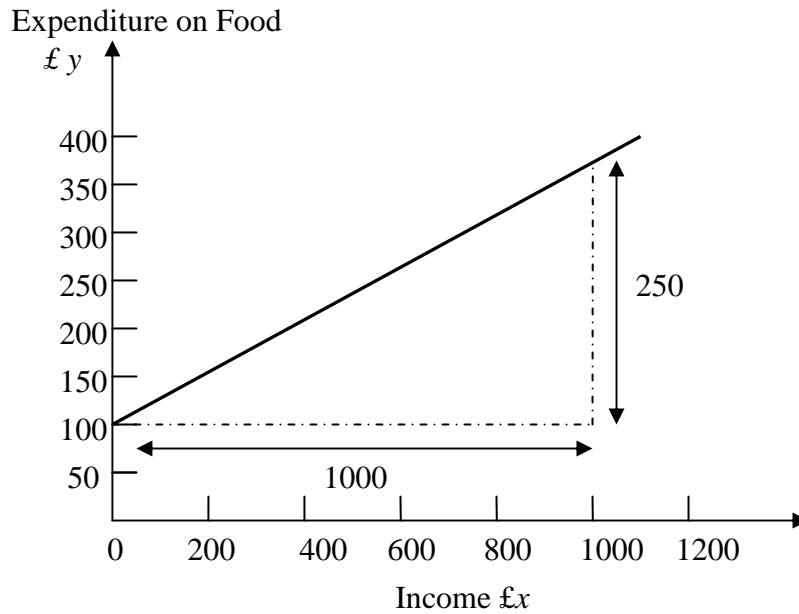
How should we interpret the intercept and slope of the line? This is simple:

The intercept, 100, tells us that if the household earns zero, it will still spend £100 per week on food (by drawing from savings or by borrowing). In economic jargon, this tells us that food is a <u>necessity</u>.

The slope, 0.25, tells us that if the household's weekly income rises by £1, its expenditure on food will rise by 25p. Alternatively, if income rises by £100, food expenditure will rise by £25. Again in economic jargon, this tells us that food is a <u>normal good</u>.

Now let's consider expenditure on a different good: public transport. This time y is now weekly expenditure on public transport in pounds; x is again weekly income. The equation which relates y to x might be:

$$y = 50 - 0.05x$$

This is a straight line with intercept 50 and slope -0.05, and is illustrated in Figure 10.
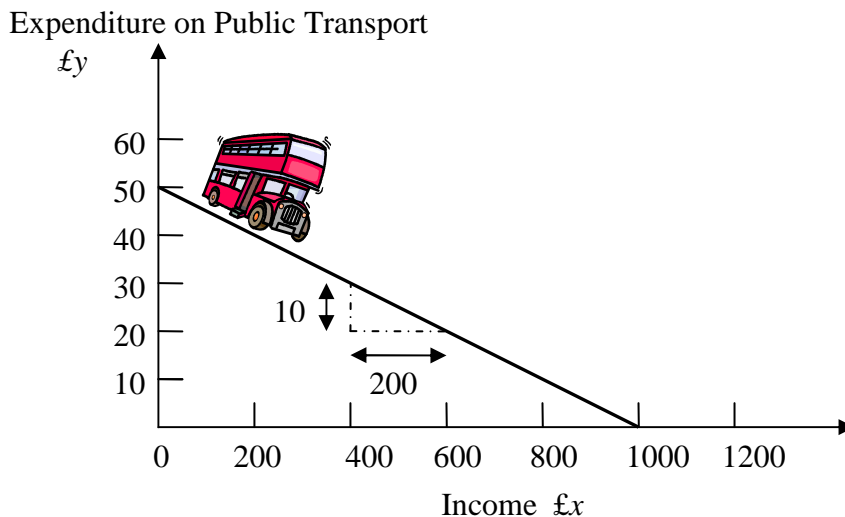
Expenditure on Public Transport

Figure 10: Engel curve for public transport

The important feature of this graph is the negative slope. As income rises, expenditure on public transport falls (because the household is substituting into private transport). Economists call this sort of good an <u>inferior good</u>.

In this case it is useful to work out the point at which the line crosses the *x*-axis, the "horizontal intercept". To obtain this, you set *y* = 0, and solve for *x*:

$$0 = 50 - 0.05x$$
$$\Rightarrow 0.05x = 50$$
$$\Rightarrow x = \frac{50}{0.05}$$
$$\therefore x = 1000$$

The importance of the horizontal intercept is that households who earn more than £1000 do not consume public transport at all.

Next, consider expenditure on TV licences. Here, the equation might be:

$$y = 2.5$$

This is shown in Figure 11, a straight line with intercept 2.5, and slope zero. In other words, it's a horizontal line at *y* = 2.5.

Figure 11: Engel curve for TV licences

TV Licence Expenditure
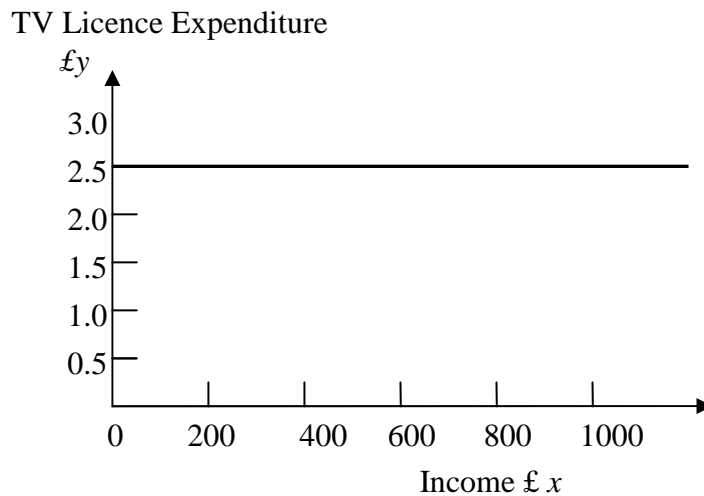
The important feature of TV licences is that every household who owns a television must have one, and the price of the TV licence is fixed, and independent of the value of the TV or the income of the household. Since nearly all households have a TV, expenditure on TV licences is effectively fixed. This expenditure is currently around £130 per annum, which averages at £2.50 per week. Hence the horizontal line at $y = 2.50$ in Figure 11. Economists call a good whose demand is not dependent on income a <u>stable good</u>.

As a final example, consider expenditure on restaurant meals. In this case, the equation might be:

$$y = -50 + 0.10x$$

This is a straight line with intercept -50, and slope 0.10. Again it is useful to work out the "horizontal intercept". Again set $y = 0$, and solve for x:
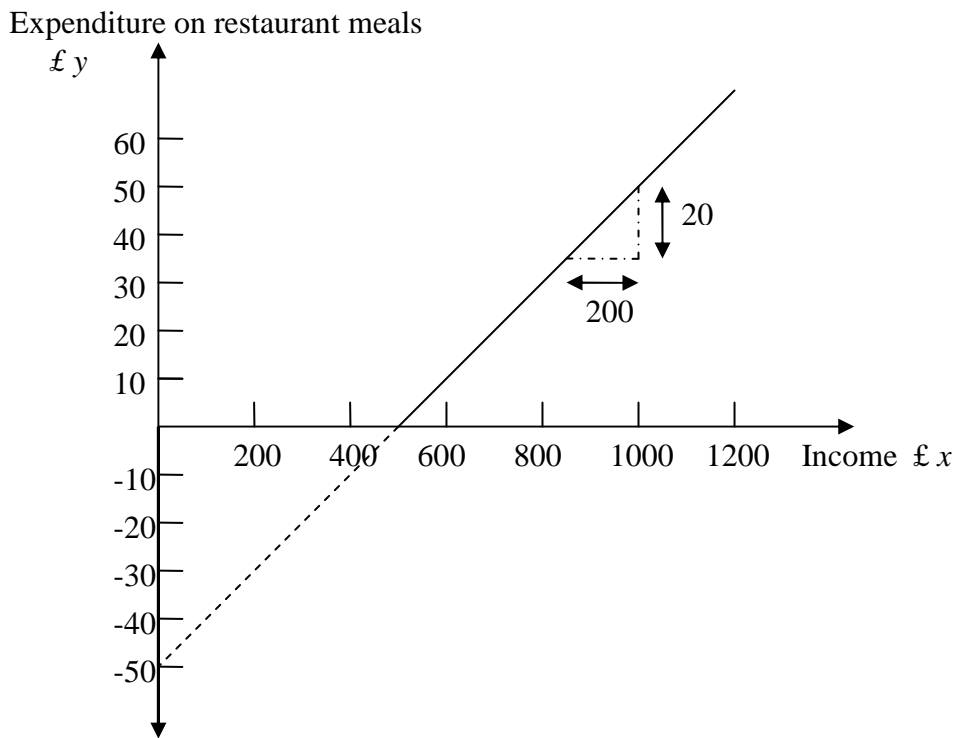
$$0 = -50 + 0.10x$$
$$\therefore 0.10x = 50$$
$$\therefore x = 500$$

The importance of the horizontal intercept in this case is that households who earn *less* than £500 per week do not consume restaurant meals at all. Economists would call this good a

luxury, since it is only consumed by households earning higher incomes. This is shown in Figure12.

Expenditure on restaurant meals

Let x be household income, and let y be household expenditure on a particular good. Consider the straight line Engel Curve:

$$y = a + bx$$

If $b > 0$, the good is a normal good.

If $b = 0$, the good is a stable good.

If $b < 0$, the good is an inferior good.

If $a > 0$, the good is a necessity.

If $a = 0$, the good is neither a necessity nor a luxury.

If $a < 0$, the good is a luxury.

The "horizontal intercept" of the line, that is, the income level at which expenditure equals zero, is given by:

$$x = -\frac{a}{b}$$