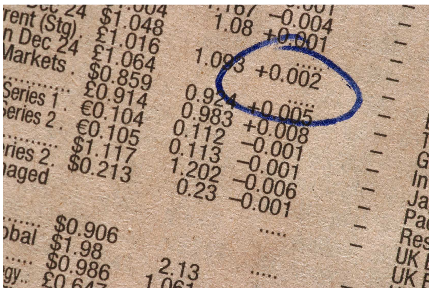

Teaching and Learning

Guide 1:

Mathematical Review

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Section 1: Introduction to the guide

This guide is designed to set out some of the basic mathematical concepts needed to effectively teach economics at undergraduate level. The basic concepts covered by this guide are: arithmetic operations, fractions, percentages, powers, indices and logarithms and the basic rules of algebra.

Economics remains a popular choice at university and continues to attract students from a wide variety of backgrounds. Some learners may choose to study several economics courses whist others may encounter economics as a complement to a business or management degree. A minority of students may choose economics as a completely different learning experience that contrasts sharply with their main academic studies.

One challenge then for economics lecturers is to ensure that all students – regardless of their prior attainment or academic discipline - are able to access economics and understand the issues and problems which it seeks to address. Since mathematics is a natural language for identifying and solving economic problems and issues, one practical problem for teaching economics is to ensure that learners are able to grasp the underpinning mathematics. Put another way, we must ensure that our learners can access the mathematics needed to access economics.

Without a good grasp of mathematical operations such as percentages, indices, fractions and logarithms, students will tend to offer mainly descriptive solutions to economic problems with little if any analytical depth. Conclusions which are drawn on purely descriptive and qualitative analysis will tend to lack the scientific objectivity and rigour which economics demands. One major challenge is to impress upon students **why** they need to be confident and competent in mathematics in order to grasp economic concepts and, more importantly, economic issues.

At the same time, there is a wider contextual issue concerning the teaching of mathematics at pre-undergraduate level. National data indicates that the popularity of A-level Mathematics has fallen significantly[1] and at the same time, the mathematical requirements of some Advanced level courses such as A-level Economics has arguably been reduced. It could be argued that the

[1] Source: Joint Council for General Qualifications. The Council reported that 67.036 students sat GCE A-level Mathematics in June 2000. The equivalent figure for June 2005 was 52.897 students; a fall of 21%.

pressure on teachers to cover the volume of A-level curriculum content might also mean that teachers have less scope to explore mathematical approaches e.g. using calculus to differentiate the teaching of marginal analysis such as marginal utility or marginal costs. An obvious implication is that students may start undergraduate courses with mathematics at Level 2 only or perhaps with some limited experience of 'higher mathematics'. This is not intended to be a criticism of students' prior attainment or of the requirements of Awarding Bodies; rather it is an observation which needs to be taken into account when planning and delivering undergraduate economics and associated courses.

Putting prior attainment to one side, there is a common issue about students' perception of their mathematical abilities. Some students will feel they have little mathematical ability and can be quite convincing in their confident assurance that they "can't do maths". Sometimes this will reflect their low confidence in using maths. Students can confuse mathematical ability with mathematical intuition: if they don't have a 'feel for numbers' they can become quite dismissive of their own capacity to learn mathematical techniques and apply them to problems. Again, this can be rooted in low 'maths confidence' and this might have been reinforced by weak performance in maths examinations.

Students with little mathematical knowledge **can** go on to understand economics; they **can** demonstrate an awareness and understanding of contemporary economic problems; and they **can** learn simple but powerful mathematical techniques to offer an independent analysis and evaluation of economic solutions.

A large part of this guide then is designed to offer practical strategies to help students achieve at economics by improving their grasp and appreciation of maths. Good learning is frequently an outcome of good teaching but high levels of subject confidence and self-esteem are also important factors. Put simply, students can do maths and apply it if they feel they can, they are shown how they can and they have opportunities to demonstrate their abilities and understanding.

This guide focuses on some of the 'mathematical fundamentals' which students need. These mathematical fundamentals translate into several skills or competencies which will mean students will have:

i) a good working knowledge of core mathematical concepts and tools;

ii) an ability to apply these concepts and tools to economic issues and problems;

iii) the capacity to think independently and critically about these concepts and know which concepts and methods to apply; and

iv) an ability to evaluate the usefulness of mathematical concepts and appreciate that mathematical tools cannot always yield perfect or complete solutions to contemporary economic problems.

The guide could be used to inform a scheme of work or to devise strategies to deliver mathematical concepts but does not need to be followed in a linear way. Some topics could be put easily together – such as fractions and percentages or arithmetic operations and the basic rules of algebra – whilst others might be delivered as discrete or 'stand alone' modules.

Section 2: Arithmetic operations

1. The concept of arithmetic operations

The label 'arithmetic operations' is quite off-putting and students may initially struggle to understand what it actually means. The essential meaning, of course, is the nut and bolts of all basic mathematics: addition (+), subtraction (-), multiplication (x) and division (÷).

All undergraduate students will recognise these operators and understand their function. Most will be able to use them confidently and with purpose whilst some may have some difficulty knowing which operator to apply in different calculations and situations. This difficulty is more common with the multiplication and division operators.

2. Presenting the concept of arithmetic operations

An effective way to deliver the concept is to start back at first principles and create a template for students to record exactly what each operator is, the different names, labels or synonyms which exist for each operator – sometimes the simple language of mathematics can confuse students - and what its function is. This might be a useful activity for students for whom English is not their first language. Typical 'operator templates' could look like this:

<table>
<tr>
<td>Definition of the operator
Addition is the process of combining two or more figures together.</td>
<td>+</td>
<td>Synonyms
Addition, adding, adding-up, summing, summation, totalling, accumulating,</td>
</tr>
<tr>
<td colspan="3">Examples of application in economics and business
1. Adding the incomes of a man and wife to calculate their joint household income
2. Adding together the spending of Government departments to calculate total Government spending
3. Summing the tax payments of all workers in an economy to calculate the total tax revenue which a Government receives over a period of time.</td>
</tr>
</table>

<table>
<tr>
<td>Definition of the operator
Subtraction is the process of removing or deducting one figure from another.</td>
<td>-</td>
<td>Synonyms
Subtraction, taking away, minus, deduction, take-off</td>
</tr>
<tr>
<td colspan="3">Examples of application in economics and business
1. Subtracting tax and National Insurance payments from a worker’s salary to calculate their net income.
2. Subtracting inflation from a pay rise to calculate the value of the real pay award.
3. Taking away the amount of money paid by a customer to a supplier to arrive at the final amount outstanding.</td>
</tr>
</table>

Definition of the operator		Synonyms
Multiplication is the process of calculating the product of two numbers.	$\times$	Multiply, times, product,

Examples of application in economics and business

1. Calculating a worker's daily wage by multiplying their hourly wage rate by the number of hours worked.
2. Calculating the revenue a firm receives from a particular product by multiplying the price of the product by the quantity sold.
3. Calculating the tax liability of a corporation by multiplying the taxable profits by the tax rate.

Definition of the operator		Synonyms
Division is the process of calculating the number of times one figure is contained in another figure.	$\div$	Divide, division, share,

Examples of application in economics and business

1. Calculating a salaried worker's hourly wage by dividing their total annual salary by the number of hours worked in a year.
2. Calculating a company's solvency by dividing current assets by current liabilities.
3. Calculating the dividend per share by dividing total profits by the number of shares in circulation.

3. Delivering the concept of arithmetic operations to small or larger groups

Larger groups could start by working through the templates (see above) and moving onto the activities (see below). Much of the learning will be secured by students actually applying their knowledge and working through the problem sets.

Smaller groups could discuss the templates as a starting point and consider some of the discussion points. Small groups could be set a simple task where they have to research three 'real world' examples where each operator is used. They could prepare and deliver a 5 minute presentation to other small groups or to the whole larger group. This activity could be extended to include simple research skills and provide students with opportunities to explore the Internet for economics and business data.

Possible sources of data include:

Web source	Description/ context
http://news.bbc.co.uk/1/hi/business/your_money/default.stm	Personal finance stories
http://www.bized.ac.uk/compfact/ratios/intro5.htm	A ratio analysis of Tesco plc
http://www.economicsnetwork.ac.uk/links/data_free.htm	Economics datasets
http://www.yfyf.co.uk/subsection.php?sID=1&ssID=1	Finance resource designed to help young people understand personal finance issues. It is aimed at 16 - 18 year olds who are considering going to work or to university.

4. Discussion questions

1. How can brackets/ parentheses completely alter the meaning of an expression or calculation?

(to reinforce the point that, for example, (100 +10)/10 = 11 whilst 100 + 10/10 = 101)

2. How can a division be calculated using only the multiplication sign?

(making the point that a/b is exactly the same as a x (1/b)

5. Activities

ACTIVITY ONE

<u>Learning Objectives</u>

LO1. Students to consolidate meaning of arithmetic operators

LO2. Students to learn how to confidently use arithmetic operators

Students are put into small groups and given laminated cards.

Three laminated cards for each number 0 -9 inclusive. Two laminated cards for each arithmetic operator +, -, x, ÷. Students given 3 laminated cards with opening bracket '(' , 3 laminated cards with closing bracket ')' and 1 '='.

The task is to agree how to arrive at given numbers using the laminated cards.

For example,

Students have: 000,111,222,333,444,555,666,777,888,999,(((,))), ++,--,xx,÷÷,=

Task 1 : Use two mathematical operators to arrive at 73

Task 2: Use all four operators to arrive at an even double digit number

ANSWERS

There are a large number of solutions and, of course, the purpose is not to create a particular solution but rather to practise the confident use of arithmetic operators.

<u>Possible solutions</u>

Task 1: 100 – 30 + 3 = 73

(5 x 3) + 58 = 73

(75 ÷5) + 58 = 73

Task 2: ((10 x 12) ÷(4 x 3)) + 34 - 26= 18

((17 + 3) x (28 – 10))÷9 = 40

ACTIVITY TWO

<u>Learning Objectives</u>

LO1. Students to consolidate basic meaning of arithmetic operators

LO2. Students to learn how to confidently use arithmetic operators

Task One

Students to complete the missing gaps from the following expressions

i) ………………… x ……………… = 225

ii) ………………… ÷ ……………… = 64

iii) (………… + ……….) / 10 = 200

iv) (………… x ……….) / (………+ 7……) = 200

v) (………… ÷……….) x (6 + …….) = 800

Task Two

Which statements are TRUE and which are FALSE?

	Tick (✓)	
	True	**False**
(a/b) = a x (1-b)		
4 + 6/5 =2		
6 + 6/2 = (6+6)/2		
13 **…….** 5 = 16.25 × **…….** The two missing gaps are ‘x’ and ‘5’ respectively		

Task Three

Complete the table below. The first row has been done for you.

Figure	Operator (+, -, ×, ÷)	Figure	Equals	Answer
5	×	31	=	155
	+	17	=	125
	÷	57.5	=	230
-4	-		=	-10
13	÷		=	
16			=	80
	÷		=	169
169	+		=	150
346	÷		=	173
567		383		

Task Four (Extension Task)
The following data is obtained from a large database of company financial information. Each company produces only one product.

	Sales (£m)	Profits (£m)	Price of product (£)	Quantity sold (millions of units)
Schumacher Ltd		45	4	25
Coulthard Plc		125	4	62.5
Wurz and Co		25	2	87.5
Alonso Plc		25	2	112.5
Massa and Co		15	12.5	24
Raikonnen		60	5	63
Montoya Ltd		10	2.5	20

Part 1
Sales are calculated by multiplying price by quantity sold.
Complete the sales column using the simple formula: Sales = Price x Quantity

Part 2
Calculate:
(a) total sales of the seven companies;
(b) total profits earned by the seven companies;
(c) total profits earned by the three most profitable companies

Part 3
Express a formula which would calculate average profit per unit sold

Part 4
(a) Which company has the highest profit per unit sold?
(b) What is the profit per unit of this company?

Part 5
A member of the research team thinks that profits should be compared against sales and decides to work out the gross profit to sales ratio. This is calculated by dividing profits by sales. Which company has the highest profit to sales ratio?

Part 6
What is the lowest profit to sales ratio?

ANSWERS

Task One

i) 15 x 15 =225, 5 x 45 =225, 9 x 25 = 255 etc

ii) 128÷2 = 64, 256÷4 = 64, 4096÷64=64 etc

iii) (1700+300)/10 = 200, (1000+1000)/10= 200, (500+1500)/10 = 200 etc

iv) (50 x 40)/(3+7) = 200, (140 x 20)/ (7+7) =200, (70 x 80)/(7+21) = 200 etc

v) (160 ÷2) x (6+4)=800, (150÷3) x (6 + 10) =800 etc

Task Two

	Tick (✓)	
	True	**False**
(a/b) = a x (1/b)	✓	
4 + 6/5 =2		✓
6 + 6/2 = (6+6)/2		✓

If: 13 ……. 5 = 16.25 x ……. The two missing gaps are 'x' and '4' respectively	✓	

Task Three

Figure	Operator (+, -, ×, ÷)	Figure	Equals	Answer
5	×	31	=	155
108	+	17	=	125
13225	÷	57.5	=	230
-4	-	**6**	=	-10
13	÷	**2**	=	**6.5**
16	×	**5**	=	80
676	÷	**4**	=	169
169	+	**-19**	=	150
346	÷	**2**	=	173
567	**+**	383	**=**	**950**

Task Four (Extension Task)
Part 1

	Sales (£m)
Schumacher Ltd	100
Coulthard Plc	250
Wurz and Co	175
Alonso Plc	225
Massa and Co	300
Raikonnen	315
Montoya Ltd	50

Part 2

(a) £1415m
(b) 305m
(c) £230m

Part 3

Average profit = Total profit/ quantity sold

Part 4

(a) Coulthard plc
(b) £2.00 per unit

Part 5

Coulthard plc (profit to sales ratio is 0.5 or 50%)

Part 6

The lowest ratio is that of Massa and Co with a ratio of 0.05 or 5%

6. Top Tips

1. Try and get students in small groups to work through the problem sets. Mixed ability groups would work best and help the most able to consolidate their understanding whilst also supporting weaker students.

2. Consider using these sessions to create opportunities for students to research economics and business data and **apply** their knowledge and understanding.

3. Students could prepare and deliver mini-presentations on arithmetic operators and the economic data they found. Students could use peer assessment, a crucial part of Assessment for Learning (AfL).

7. Conclusion

Students will need to practise the basic arithmetic operators in order to feel confident but also so they can really understand what the applied function of each is. If they have a good understanding of the function of each and have had opportunities to practice these they should be able to attempt more complex problem sets. Given the wide range of confidence and competence that lecturers are likely to see in their classes, it is also important to try and differentiate the material as much as possible: some students will need a quick recap whilst others will need more fundamental and comprehensive teaching.

Section 3: Fractions

1. The concept of fractions

Fractions are often seen as an abstract and 'old' topic by students many of whom will not have a clear understanding of why they need to learn it. Most students will easily understand 'nice fractions' such as ¼,½,¾, or ⅛ but will struggle with more unusual fractions such as 14/15 or 23/56. Students may not understand or feel comfortable with vulgar or improper fractions such as 21/15 and sometimes they can interpret improper fractions simply as 'errors' because they do not expect the numerator to be larger than the denominator.

2. Presenting the concept of fractions

An obvious way to improve students' learning of fractions is to show them **why** fractions are important. Putting the concept into a practical context will help students to access the concept and enhance learning. The video clip on "Fractions and Decimals" would provide a useful starting point.

A simple way is to provide a summary of how fractions are used in the world of economics and business. This could be created with a montage of images and economics news articles or by asking students to research this for themselves.

stock prices.xls

	A	B	C	D
1				
2	Closing Price Per Share			
3	January	14 1/8	14 2/16	
4	February	15 3/8	15 6/16	
5	March	16 1/4	16 4/16	
6	April	14 3/16	14 3/16	
7	May	11 1/4	11 4/16	
8	June	9 7/8	9 14/16	
9	July	12 1/8	12 2/16	
10	August	13 3/8	13 6/16	
11	September	15 5/16	15 5/16	
12	October	15 3/8	15 6/16	
13	November	18 7/8	18 14/16	
14	December	20 3/4	20 12/16	
15				

Sheet1

Typical examples could include:

- calculating gambling odds and other risk returns e.g. England quoted at 7/1 to win the World Cup;

- calls for UK interest rates to be cut by ¼%;

- the Bank of England inflation target is 2½% per annum;

- penny share prices (see left);

3. Delivering the concept of fractions to small or larger groups

Students could be asked to research ways in which fractions are used. Small groups could look at penny shares for example or betting odds. What were the odds on Brazil to win the World Cup in 2006 and what might someone have won if they had wagered, say, £100. Larger groups could perhaps view a Powerpoint presentation on “Contemporary Fractions” which had been researched and delivered by a peer group

Links with the online question bank

The online question bank incorporates questions on fractions and this can be found on the METAL website at:

http://www.metalproject.co.uk/METAL/Resources/Question_bank/Numbers/index.html

There are many questions which students can work through independently.

Video clips

There are two relevant video clips which can be found at http://www.metalproject.co.uk/Resources/Films/Mathematical_review/index.html

Clip 1.01 “Understanding Fractions – Introduction” gives a very good overview of the importance of fractions. The second clip 1.02 entitled “Multiplying Fractions – Stonehenge” uses fractions to better understand a construction related problem.

4. Discussion questions

Discussion questions could flow naturally from the video clips and could help to create a plenary session.

1. Explain how fractions can help to solve a problem.

2. Fractions can be used to calculate risk but what is the difference between risk and uncertainty? In small groups, identify some things which have known risk and for others where there is likely to be considerable uncertainty.

5. Activities

ACTIVITY ONE

LO1: Students learn how to calculate simple fractions

Task One
Complete the database below using this information.

	Price (£ per kilo)	Cost of 3 kilos	Cost of 3 kilos if price is halved
Apples	2.00		
Bananas	3.50		
Cherries	4.00		
Dates	6.00		
Figs	2.45		
Gooseberries	1.25		
Kiwi	2.90		
Loganberry	3.50		
Plums	4.15		
Raspberries	5.00		
Strawberries	6.30		
Watermelon	1.50		

Task Two
How much would 6 kilos of dates cost using the original prices?

Task Three
If the price of apples halves, what is the difference in total cost for 3 kilos of apples?

Task Four
Express the cost of 3 kilos of apples as a fraction of the cost of 3 kilos of dates. Use the original prices.

ANSWERS

Task One

	Price (£ per kilo)	Cost of 3 kilos	Cost of 3 kilos if price is halved
Apples	2.00	6.00	3
Bananas	3.50	10.50	5.25
Cherries	4.00	12.00	6
Dates	6.00	18.00	9
Figs	2.45	7.35	3.675
Gooseberries	1.25	3.75	1.875
Kiwi	2.90	8.70	4.35
Loganberry	3.50	10.50	5.25
Plums	4.15	12.45	6.225
Raspberries	5.00	15.00	7.5
Strawberries	6.30	18.90	9.45
Watermelon	1.50	4.50	2.25

Task Two
£36.00

Task Three
The apples cost £3.00 less.

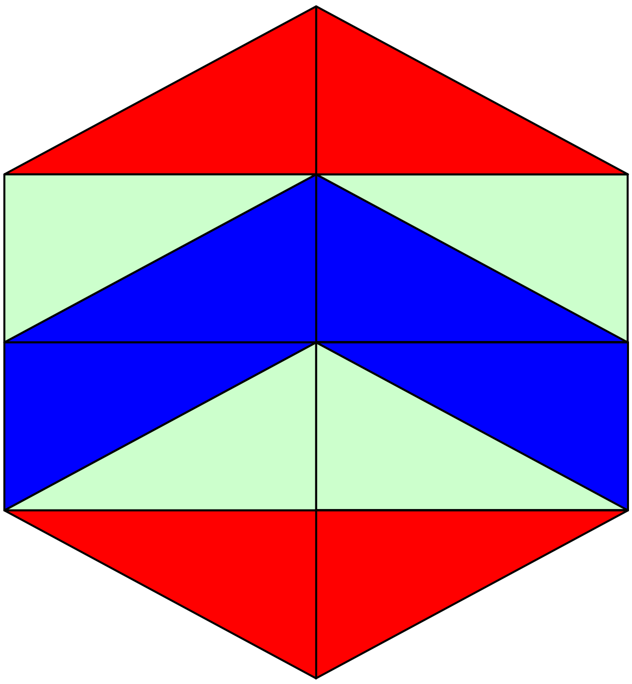
Task Four
¼

ACTIVITY TWO

Learning Objectives

LO1: Students learn how to calculate simple fractions
LO2: Students learn how to apply fractions to simple probability tasks

The owner of a large fairground is considering whether to introduce a new game. The game is essentially a variation of darts. The board is made of 12 identically sized triangles and looks like this:



Each triangle is right angled and has a base of 10cm and a height of 6cm.

Task One
Calculate the total area (in cm^2) of:
(a) all of the blue triangles; and
(b) the area of the board.

Task Two
What fraction of the board is red?

Task Three
If a contestant hits a red or blue area he wins £5.
(a) if a contestant successfully hits the board, what is the probability that they will win £5?

(b) if, on average, a contestant hits the board once in two attempts, what is the likelihood they will hit a blue area on their first attempt?

ANSWERS

Task One
(a)120cm^2
(b) 360cm^2

Task Two
1/3

Task Three
(a) 8/12 or 2/3
(b) ½ x 1/3 = 1/6

6. Top Tips

Ask students to think about what a fraction says e.g. what does "¼" actually mean? What is the "1", what is the "4"? Ask students to research "fractions in Economics and Business" and then get them to apply this to the problem sets.

7. Conclusion

Fractions are a simple but effective way to describe proportions and amounts. An effective introduction to fractions could lead neatly into percentages. The two concepts could be taught together or, at least, sequentially.

Section 4: Percentages

1. The concept of percentages

A percentage is a simple but powerful concept which allows a part or a proportion to be expressed as a fraction of 100. Students can quickly learn how to accurately calculate a percentage but they might struggle to explain what percentage is actually stating. That is, the distinction between a “rote-calculation” and a “mathematical inference” can be elusive.

The video clip “Percentages” (see below)would provide a good overview of percentages and links fractions with percentages. Students could consider situations where they might encounter percentages and then watch the video clip to consolidate their knowledge and understanding.

One strength of using percentages is that proportions and fractions can be easily compared because they are converted to the same base i.e. 100. For example, in the simple example below we can see who pays the most tax (in crude or absolute terms) but it is not easy to see which person is being **taxed** the most because we cannot easily infer what the different tax proportions are.

Example: Using percentages to understand the tax burden

Students could consider the two facts and question below:

FACT 1: Worker A earns £20,000 and he pays £6000 in tax.

FACT 2: His colleague person B earns £45,000 and she pays £9000 in tax

QUESTION: Who is being taxed the most?

From these figures we cannot easily see which person is taxed the most given their income. Put another way, we need a quick way to calculate **the tax burden**. This is shown in the simple calculation below

<u>Calculating tax as proportion of earned income</u>

Person A pays tax equal to (£6,000/£20,000) x 100% =30%

Person B pays tax equal to (£9,000/£45,000) x 100% = 20%

The fact that person B pays more tax in terms of pounds Sterling (£) is more than offset by the fact that they earn considerably more than person A. In effect, the percentage calculation translates the salaries into a common base of 100 and then expresses the tax paid as a proportion of the base. Because we have a common base – the figure of 100 – we can compare the tax paid on a like-for-like basis. In this case, person B pays more “tax pounds” but taking into their salary, we can see she is actually taxed at a lower rate for each pound earned than her colleague

2. Presenting the concept of percentages

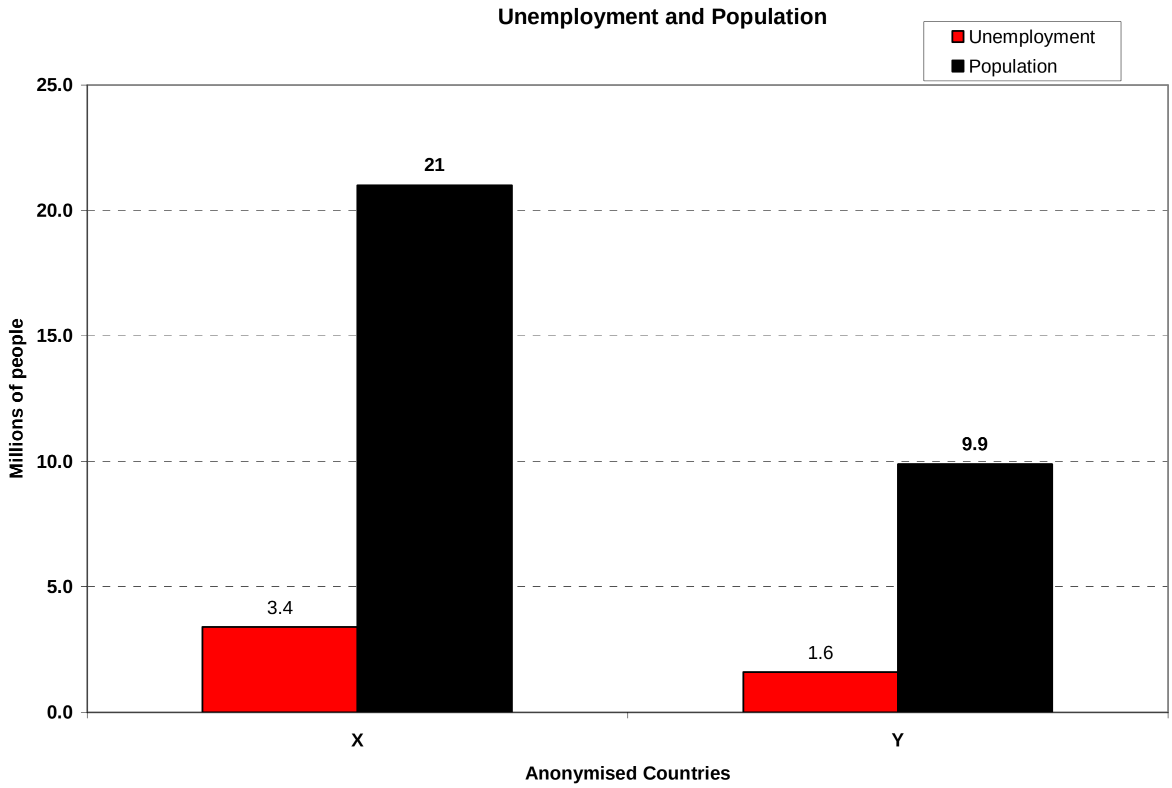
Students can often struggle with percentages. Some may say they “cannot do percentages” and others probably feel they should know what a percentage means and even if they can calculate one they cannot quite articulate what a percentage says. The trick is to initially teach percentages as fractions or parts by going right back to fundamentals.

Students are more likely to learn if they understand why they are learning a particular topic or concept. One good method of introducing percentages would be to explain the practical application of percentages such as:

- easier to show changes in fractions and proportions e.g. has the share price of British Airways performed better or worse than an average company in the FTSE-100?;
- helps to show relative rather than absolute changes e.g. is the gap between rich and poor getter wider or more narrow?; and
- can help to express trends and longer term movements e.g. how has the Stock Market performed?;

The usefulness of percentages can also be presented by using a simple graphical technique. This approach would also cater for visual learners and reinforce the difference between absolute and relative change.

Look at graph A which shows unemployment in two nations (X) and (Y). Which country has the highest **unemployment rate**?



It seems that Country X would have the higher unemployment rate. In fact, if the unemployment figure is divided by the population for each country, we get exactly the same unemployment rate.

That is, 3.4/21 = 1.6/9.9 = 16% for both nations.

3. Delivering the concept of percentages to small or larger groups

Small group work could be an ideal way for students to practise the calculation and interpretation of percentages. It would also create opportunities for discussion and help students to improve their 'intuitive grasp' of the concept.

Some possible activities for smaller groups could include:

- matching exercise using laminated cards (see Activity section below);

- asking students to use the internet or newspapers to find examples of percentages and prepare a brief 5 minute presentation on:

i) defining what is meant by a percentage;

ii) explaining how they think the percentage figure was calculated by the writer of the article; and

iii) what the percentage figure tells them.

Larger groups could also undertake the presentations with students presenting to each other in smaller groups. Students could offer peer assessment on the strengths of the presentation and offer feedback on the accuracy of the answers and the clarity of the exposition. In this way, students would use Assessment for Learning techniques. The lecturer would need to have a feel for the ability of class members in order to create 'blended groups' which would reflect the range of abilities.

Links with the online question bank

The online question bank contains many applied questions involving percentages. Straightforward calculations of percentages can be practised using the questions at http://www.metalproject.co.uk/METAL/Resources/Question_bank/Economics%20applications/index.html

These can then be consolidated with calculation of market shares at: http://www.metalproject.co.uk/METAL/Resources/Question_bank/Economics%20applications/index.html

and students could also apply their understanding to problems involving the calculation of interest at:

http://www.metalproject.co.uk/METAL/Resources/Question_bank/Economics%20applications/index.html

Video clips

The video clip can be found at:

http://www.metalproject.co.uk/Resources/Films/Mathematical_review/index.html

There are three clips that students will found useful. The first clip is labelled as "1.03 Percentages - Car Prices" and focuses on the method of working out percentages using the example of depreciation. This is extended with clip "1.04 Percentages – the Big Mac Index" which colleagues will recognise from The Economist. The third clip provides a plenary and covers fractions and powers too. This is entitled "1.06 Fractions, Percentages, and Powers Conclusion".

4. Discussion questions

i) Why are percentages useful when comparing economic statistics from different countries?

ii) Why might changes in small numbers produce (misleadingly) high percentage changes?

iii) Why is a percentage often easier to understand than a fraction?

iv) Using video clip 1.04 at:

http://www.metalproject.co.uk/Resources/Films/Mathematical_review/index.html

explain why an economist would want to calculate the Big Mac Index. How might this simple economic measure help a corporation decide where to locate a new factory in the world? A simple extension of this would be to ask groups to consider why large textiles factories locate in South East Asia and what economic information would firms such as Nike, Adidas and Reebok have used to make this decision.

5. Activities

ACTIVITY 1

<u>Learning Objectives</u>
1) Students understand the mapping between fractions and percentages
2) Students can independently calculate simple percentages
3) Students can apply percentages to a given problem

Consider the table below. Draw a line between each fraction and the equivalent percentage.

Fraction
1/10
½
1/3
1/4
1/6
1/7
2/3
4/6
5/7
9/10
15/20
7/14
23/100

Percentage
71.4%
50%
25%
66.6%
16.6%
50%
14.2%
90%
75%
33.3%
23%
10%
66.6%

ANSWERS

Fraction
1/10
½
1/3
¼
1/6
1/7
2/3
4/6
5/7
9/10
15/20
7/14
23/100

Percentage
71.4%
50%
25%
66.6%
16.6%
50%
14.2%
90%
75%
33.3%
23%
10%
66.6%

ACTIVITY TWO

This activity could be preceded by the video clip (Field 1.E.2) which sets a strong and applied macroeconomic tone.

<u>**Learning Objectives**</u>

1) Students to understand basic macroeconomic data

2) Student to be able to calculate percentages and relate numerical data to graphical illustrations

An economist collates some basic data on a number of countries. This is summarised in the tables below for two years 1995 and 2005.

Data for 1995

	Unemployment (millions)	Inflation (% per annum)	Population (millions)	GDP (£ billions)
Country A	5	1	125	2125
Country B	3	3	100	1250
Country C	2	2	57	1311
Country D	13	4	250	5000
Country E	6	1	165	2640

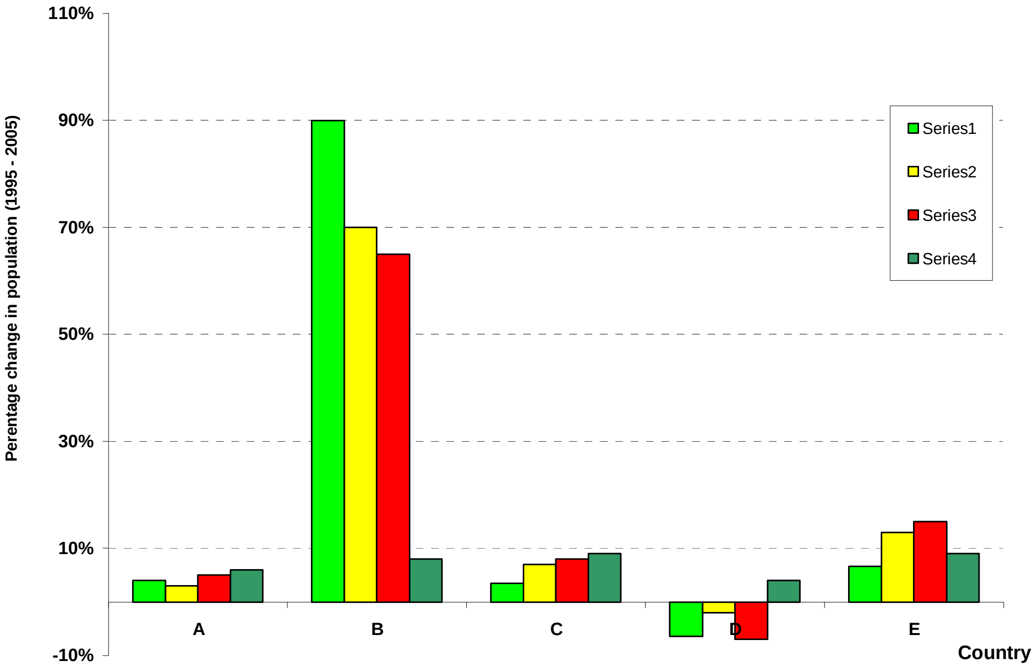
Data for 2005

	Unemployment (millions)	Inflation (% per annum)	Population (millions)	GDP (£ billions)
Country A	4	2	130	2125
Country B	3	4	190	1250
Country C	3	1	59	1311
Country D	12	6	234	5000
Country E	8	3	176	2640

Task 1
(a) Which country experienced the biggest positive change in unemployment?
(b) What was the percentage change?

Task 2
Which country had the biggest unemployment rate (unemployment as a proportion of the total population) in 1995 and what was the figure?

Task 3
Look at the graph below. Which series (Series 1,2,3 or 4) illustrates the percentage changes in population (1995 to 2005)?



ANSWERS

Task 1
Country C with 50% increase in unemployment

Task 2
Country D with 5.2%.

Task 3
Series 1(green bars)

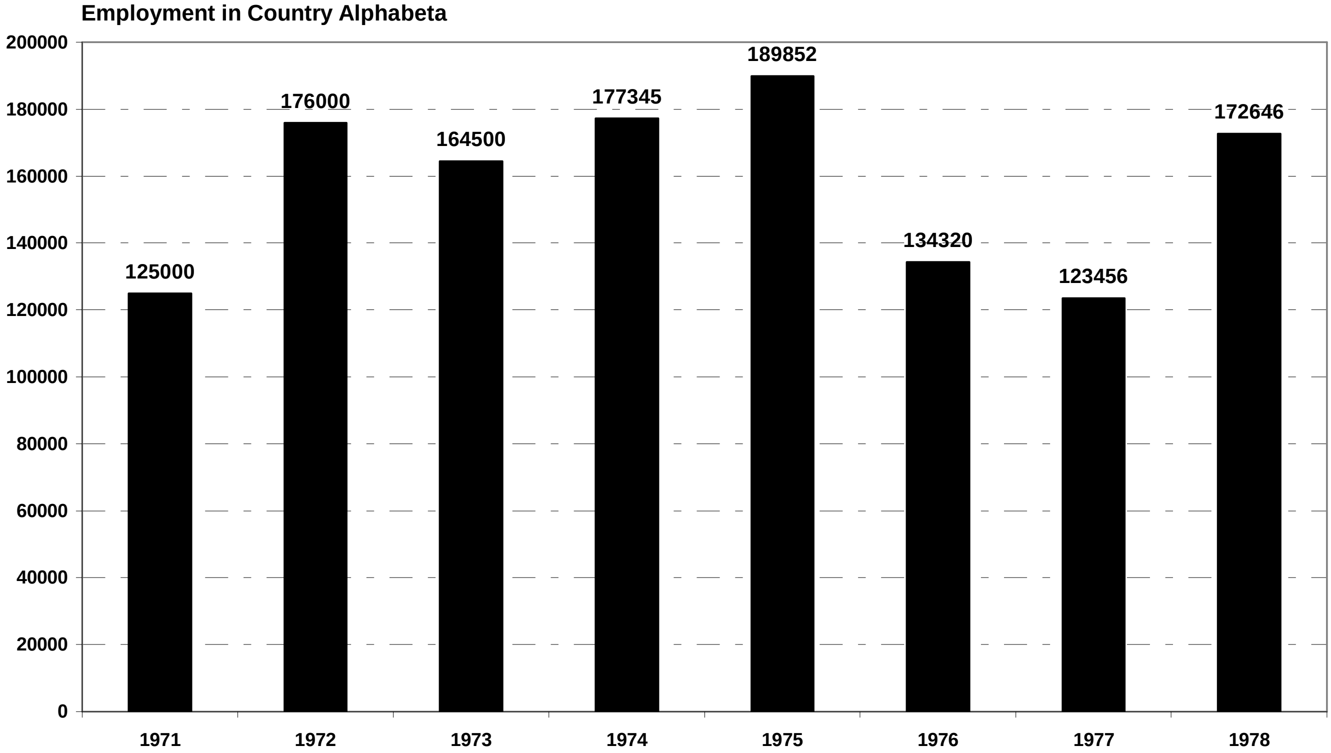
ACTIVITY 3

<u>Learning objectives</u>

LO1. Students to be able to calculate simple percentages
LO2. Students to be able to calculate changes in percentages, and make simple inferences.
LO3. Students to be able to use percentages in simple "What..If" scenarios

An economist researches Country Alphabeta and finds that employment in the country during the 1970s varied considerably from one year to the next.

He chooses to show his data graphically in a brief research paper. His graph is shown below.



He sends his paper to a colleague for comments. His friend says that the graph is difficult to interpret and poses a number of questions. Your task is to answer these on behalf of the researcher.

Question 1
Using the above data produce a table showing the percentage change in employment year on year?

Question 2

(a) Calculate the change in employment in terms of the number of people employed from the start point in 1971 to the end point in 1978

(b) What is the average percentage change in employment from 1971 to 1978?

(c) What would employment be in 1979, 1980 and 1981 if the average percentage change calculated in (b) continued?

Question 3
The researcher is told that his research is incorrect and that in fact:

i) employment was indeed 125,000 in 1971; and
ii) employment rose each year by exactly 2.5%.

What would the employment figure for 1978 be?

ANSWERS

Question1

1971	-
1972	+40.8%
1973	-6.5%
1974	+7.8%
1975	+7.1%
1976	-29.3%
1977	-8.1%
1978	+39.8%

Question 2

(a) =172646 – 125000= **47646**

(b) =((172646-125000)/125000)* 100% = **+38.1%**

(c)

Year	Employment
1971	125000
1972	176000
1973	164500
1974	177345
1975	189852
1976	134320
1977	123456
1978	172646
1979	**238424**
1980	**329264**
1981	**454713**

Question 3

Employment figure would be 125000 x $(1.025)^7$ = 148586

6. Top Tips

1. Start teaching or refreshing percentages by reviewing more fundamental questions e.g. What is a fraction? What is a proportion? The video clips (see above) give a clear overview of these concepts.

2. Explain **why** percentages are important and refer to the 'applied video clips' (see above). The clips could be supplemented by newspaper cuttings which are a good source of relevant and topical information e.g. inflation rates, crime rates, changes in asylum applications etc. This will help students to realise the importance of percentages.

3. Ask students to rate their confidence in calculating and interpreting percentages at the beginning of the course. For example, using a simple 1 -10 scale. Alternatively, state an assertion followed by a Likert scale such as:

	Please tick (✓)			
	Strongly agree	**Agree**	**Disagree**	**Strongly disagree**
I understand the meaning of a percentage.				
I can confidently calculate a percentage using given data.				
I can interpret a percentage and make sense of what it says.				

Repeat the survey at an appropriate time – at the end of a block of teaching perhaps – and show the students their individual responses and group responses. This will hopefully show that their competency and confidence has improved and given an indication of the 'distance travelled'.

7. Conclusion

There are three key factors when teaching percentages. First, there needs to be an initial emphasis on the basics of fractions and proportions. Second, students need to have opportunities to calculate and interpret percentages and to be able to comment critically on the usefulness of percentages. Third, students need to have a **confident** grasp of percentages and their initial fears or low esteem of their mathematical ability needs to be addressed.

Section 5: Powers

1. The concept of powers

Many students will recognise simple powers such as squared or cubed e.g. 3^2 or 5^3 and many will have used these in level 1 or level 2 maths papers e.g. calculating surface areas or volumes. A minority will have used these in a more complex way e.g. calculating volumes of revolution. One challenge will be to differentiates this material adequately so all students are able to access the material but also stretch their learning.

A simple way to introduce the concept of powers would be to first explain the meaning and significance.

For example,

$$3^4$$

The base figure

The figure ‘4’ is the power or ‘replicator’

A power describes or states by how many times the base figure should be multiplied by itself. Thus,

$$3^4 = 3 \times 3 \times 3 \times 3$$

2. Presenting the concept of powers

An effective way to present this concept is to demonstrate its usefulness: the better the context the more likely students will learn how the ‘power tool’ might be used.

Typical economics and business examples where powers can be used include:

- calculating compounded figures such as interest payments on a mortgage;
- working out areas or volumes e.g. how much revenue BP can expect to earn will partly depend on the volume of oil it can pump out of its global oil and gas fields;
- estimating the expected return from an investment project (net present values); and
- calculating the value of an individual’s savings or pension in times of high and sustained inflation.

Students will need help learning the basic rules of powers and indices, namely:

① $\alpha^n = \alpha_1 \times \alpha_2 \times \alpha_3 \times \ldots . \alpha_n$

② $\beta^{-n} = 1/\beta^n$

③ If: $\alpha^\beta = \gamma$, then:

$\beta \times \ln\alpha = \ln\gamma$

Where $\ln\alpha$ is the natural logarithm of α

3. Delivering the concept of powers to small or larger groups

Small groups and student pairs could work through simple task sheets (see Activity One below) and discuss which statements are true/false and why. This would strengthen team working and peer assessment as students discussed their reasoning. Small groups could be encouraged to research examples of powers and indices in the media and to report back what they found.

Students could explore websites and identify what the main purpose or activity of the organisation is. The lecturer could then explain how indices and powers could be used by that organisation.

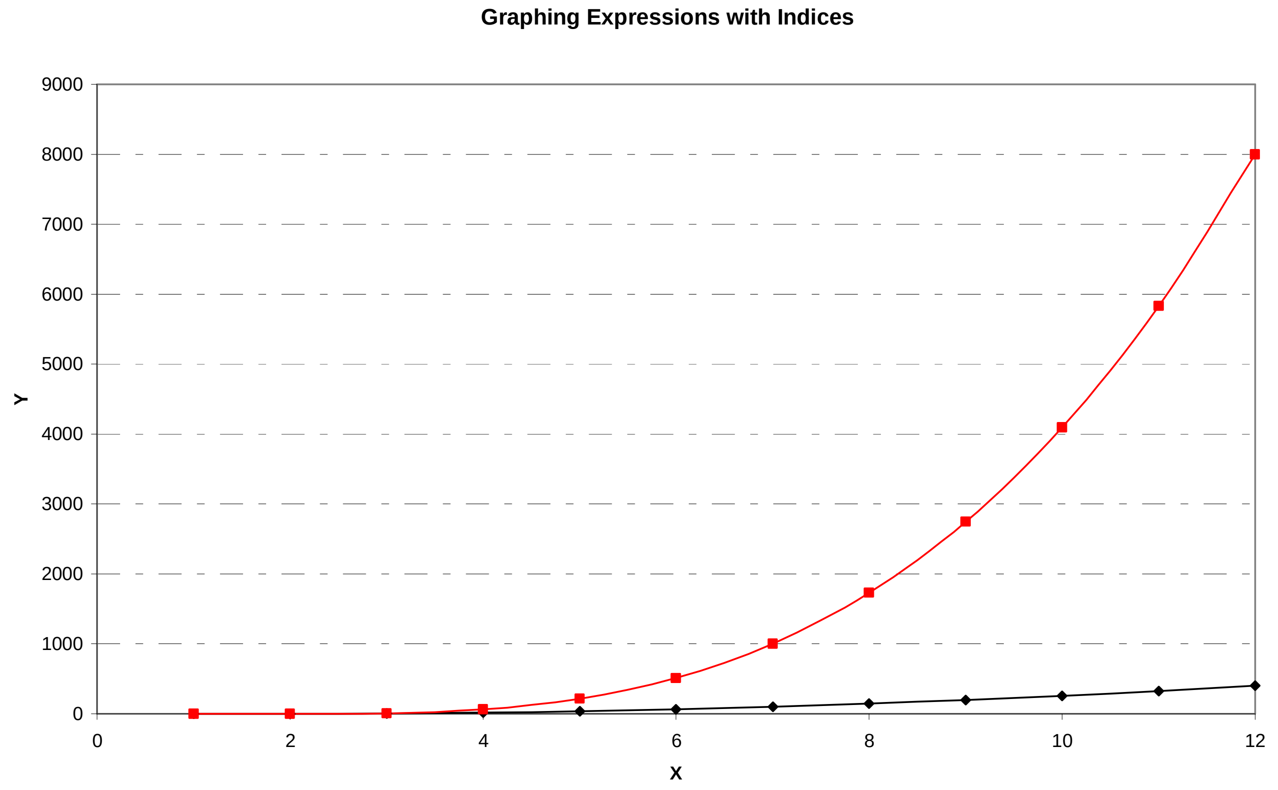
For example, in the template below students would research and complete columns ①② and ③. When feeding back what they found the lecturer would explain in column ③ the usefulness of powers and indices. The first row has been completed as an example.

Organisation	Website	Sector ①	Primary Business activity ②	Why would powers and indices be useful ③
3i	www.3i.com	*Financial Services*	*Financial investment in business*	*To calculate the future streams of expected profits in today's money (discounted cash flow)*
Prudential	http://www.pru.co.uk/save_invest/			*To calculate the real value of a future pension*
BP	www.bp.com			*Calculate the volume of oil and gas which can be brought ashore (volume of the pipeline capacity) $V = \pi r^2 L$ where* *V= Maximum capacity of pipeline* *r = radius of pipeline* *L = length of pipeline*
Ford	www.ford.com			*How much metal will be needed to produce wheels?*
GlaxoSmithKline	www.gsk.com			*Measuring the growth rate of bacteria and viruses.* *Calculating volumes of chemicals needed for large-scale medicine production.*

Larger groups would probably benefit more from formal instruction on the 'rules' of powers/indices and this would then lead them into the tasks. Larger groups of students could be shown images of particular indices and asked to match to given expressions. This could help students to grasp a more intuitive feel for powers and indices.

For example,

Large groups could consider the following simple slide and decide which graph represents $y = x^{3;}$ the red line or the black line?[the answer is of course the red line; the black line graphs $y=x^2$]



Links with online question bank

The online question bank offers a variety of questions which test students' understanding of indices. Questions on 'pure indices' can be found at http://www.metalproject.co.uk/METAL/Resources/Question_bank/Algebra/index.html

Higher level and more applied material is also provided with questions on investment appraisal at :

http://www.metalproject.co.uk/METAL/Resources/Question_bank/Economics%20applications/index.html

although this is covered Teaching and Learning Guide 5 on Finance and Growth.

Video clips

The video clips can be found at:

http://www.metalproject.co.uk/Resources/Films/Mathematical_review/index.html

The clip “1.05 Powers - US Merger Policy” provides a good example of why powers are useful in competition policy with reference to the Herfindahl index and this is also summarised in the plenary clip “1.06 Fractions, Percentages, and Powers Conclusion”. Higher ability students could watch the clip “1.09 Powers - Production Functions in Gold Mining and Oil Production” which directly maps powers to costs and outputs, especially if they have covered this microeconomic material.

4. Discussion questions

1. What is the link between powers/indices and rates of growth?

2. How could an index/power be used to calculates rates of decrease/ diminution

3. What does a fractional power mean e.g. $x^{2/3}$?

4. How can indices help economists to fit lines of best fit?

[Tip: could consider phenomena such as business cycles and look at GDP data and how only a polynomial expression could provide a reasonable regression]

5. Activities

ACTIVITY ONE

<u>Learning Objectives</u>

LO1 : Students learn how to use simple calculations using powers

LO2 : Students learn how to manipulate expressions involving powers

Task One

Work out the following expressions using simple powers:

Expression	Answer
3^3	
5^3	
6^4	
6^6	
7^3	
11^3	
4.5^3	
2.2^2	
$4.4^{4.4}$	

Task Two

In some way, powers or indices are similar to multiplication and division.

One number (A) may be divided by another number (B) to calculate an answer (C). Alternatively, A may be multiplied by the reciprocal of B to calculate C

Put another way,

$A \div B \equiv A \times 1/B = C$

Similarly, powers can be used in a 'symmetrical way'. A negative power can be expressed as a positive power when it is the reciprocal

$3^{-2} = 1/3^2 = 1/9$

$4^{-3} = 1/4^3 = 1/(4x4x4) = 1/64$

$5^{-5} = 1/5^5 = 1/(5x5x5x5x5) = 1/3125$

Using this knowledge, answer the following questions:

Expression	Tick one of the columns	
	True (✓)	False (✓)
$2^2 = 4$		
$2^2 = -4$		
$3^3 = 9$		
$3^4 = 18$		
$3^{-2} = 1/9$		
$4^{-2} = 1/16$		
$10^2 = 4.64^3$		
$9^2 = 4^3$		
$12^3 = 36$		
$8^3 = 2^9 = 2 \times 16^2$		
$6561 = 81^{-2}$		
$4^{-2} = 16^{-1}$		
$12^{-3} = 2^{-2} \times 432^{-1}$		

Task Three

An economist creates an expression – known as a "production function" which describes how capital (K) and labour (L) can be combined to create output (Q).

If:

$Q=K^{\alpha}L^{(1-\alpha)}$

(a) Complete the table:

α	K	L	Q
0.1	**0**	**100**	
0.1	**10**	**90**	

0.1	**20**	**80**	
0.1	**30**	**70**	
0.1	**40**	**60**	
0.1	**50**	**50**	
0.1	**60**	**40**	
0.1	**70**	**30**	
0.1	**80**	**20**	
0.1	**90**	**10**	
0.1	**100**	**0**	

(b) What do you notice about the powers of α and β?

(c) If:

$Q = K^{\alpha}$ then complete the tables:

	α	**Q**		**α**	**Q**		**α**	**Q**
	0	1		0	+1		0	1
	1	5		1	-1		1	0.1
K= 5	2	25	**K=-1**	2	+1	**K=0.1**	2	0.01
	3	125		3	-1		4	0.0001
	4	625		4	+1			
	5	3125		5	-1			
	6	15625		6	+1			

(d) In each case, rearrange the expression to make 'α' the subject:

i) $Q=K^{\alpha}$

ii) $3Q=4K^{\alpha}$

iii) $3K= 2Q^{\alpha}$

iv) $3\alpha^2=Q/K$

ANSWERS

ACTIVITY ONE

Task One

Expression	Answer
3^3	27
5^3	125
6^4	1296
6^6	46656
7^3	343
11^3	1331
4.5^3	91.13
2.2^2	4.84
$4.4^{4.4}$	677.94

Task Two

Expression	Tick one of the columns	
	True (✓)	**False (✓)**
$2^2 = 4$	✓	
$2^2 = -4$		✓ $2^2 = 4$
$3^3 = 9$		✓ $3^3 = 27$
$3^4 = 18$		✓ $3^4 = 81$
$3^{-2} = 1/9$	✓	
$4^{-2} = 1/16$	✓	
$10^2 = 4.64158^3$	✓	

$9^2 = 4^3$		✓ $9^2 = 81$ $4^3 = 64$
$12^3 = 36$		✓ $12^3 = 1728$
$8^3 = 2^9 = 2 \times 16^2$	✓	
$6561^{-1} = 81^{-2}$	✓	
$4^{-2} = 16^{-1}$	✓	
$12^{-3} = 2^{-2} \times 432^{-1}$	✓	
$6^{-6} \times 3^2 \times 4^2 = 1/300$		✓ $6^{-6} \times 3^2 \times 4^2 = 1/324$

Task Three

(a)

α	K	L	Q
0.1	0	100	0.00
0.1	10	90	72.25
0.1	20	80	69.64
0.1	30	70	64.31
0.1	40	60	57.62
0.1	50	50	50.00
0.1	60	40	41.66
0.1	70	30	32.65
0.1	80	20	22.97
0.1	90	10	12.46
0.1	100	0	0.00

(b) The powers or indices of α and β sum to 1. This particular production function is a special case: the Cobb-Douglas function.

(c)

K= 5	α	Q	K=-1	α	Q	K=0.1	α	Q
	0	1		0	+1		0	1

1	5	1	-1	1	0.1
2	25	2	+1	2	0.01
3	125	3	-1	4	0.0001
4	625	4	+1		
5	3125	5	-1		
6	15625	6	+1		

(d) i) $\alpha=\ln Q/\ln K$

ii) $\alpha=\ln 3Q/\ln 4K$

iii) α-$\ln 3K/\ln 2Q$

iv) $\alpha=\sqrt{(Q/3K)}$

ACTIVITY TWO

Learning Objectives

LO1 : Students learn how to use indices to calculate simple compound interest

Task One

Jean Cheesman invests £10,000 in an interest bearing account. The bank will pay her 3% on the balance she has in the account at the end of the year.

What simple expression would calculate her:

(a) bank balance(B) after n years [assumes she withdraws none of her money]?

(b) the interest she receives after n years

(c) the **real** value of her interest if inflation is always two thirds of the rate of interest?

(d) What would the rate of interest need to be if Jean Cheesman expected her savings balance to be £17,000 after 6 years?

(e) How many years, to the nearest whole year, would Jean Cheesman need to invest her money if:

- her principal sum was still £10000;
- the rate of interest was 2.3%; and
- she wanted to have at least £19,750?

ANSWERS

Task One

(a) $B = 10{,}000 \times 1.03^n$

(b) $i = (B - 10{,}000)$ or, $10000(1 - 1.03^n)$

(c) Real $i = 10000(1-1.01^n)$

(d) $i = \sqrt[6]{(1.7)} = 1.09$ i.e. 9% per year for each year

(e) $n = \ln(19750/1000)/\ln(1.023) = 29.93$ years or 30 years to the nearest whole year.

6. Top Tips

The idea of powers and indices could be quite difficult for students to grasp, beyond simple squaring and cubing. Ironically, it might be effective to start with discussion questions first; to get students thinking about how powers and indices are useful and how they can be applied to solve economics problems.

A simple overview of the 'rules' might be useful too, perhaps with a simple summary sheet such as:

Expression	**Alternative expression using indices rules**
$\alpha^{\beta}=\gamma$	$\beta\ln\alpha = \ln\gamma$
$\alpha^{-\beta}=\gamma$	$1/(\alpha^{\beta}) = \gamma$
$\alpha^{\beta}=\gamma$	$\alpha = \sqrt[\beta]{(\gamma)}$ where $\sqrt[\beta]{\ }$ is the βth root.

7. Conclusion

Powers and indices are crucially important to economic calculations such as growth, compounding and rates of change. Students need to have a good applied understanding of indices to support and extend their conceptual understanding. Students should be given plenty of practice applying the simple rules of indices.

Section 6: Logarithms

1. The concept of logarithms

Indices and logarithms are a very useful concept in economics and business and although some will have used indices before, very few will have encountered logarithms before.

2. Presenting the concept of indices and logarithms

A good way to introduce the concept is by referring to the video clips. The video clips can be found at http://www.metalproject.co.uk/Resources/Films/Mathematical_review/index.html

The clip "1.07 Logarithms and Population Growth" would help to set the scene very well and demonstrate why logarithms are useful and important. This clip is based on population growth and this could be extended by asking students to select a country whose population they would like to research. A mapping package such as Google Earth could help to stimulate students' interest. Similarly, the CIA website offers a large volume of international data http://www.cia.gov/cia/publications/factbook/index.html that could help 'springboard' students into this topic.

3. Delivering the concept of indices and logarithms to small or larger groups

Lecturers might find it useful to link this with the section on powers and indices since the two topics are closely linked. Small groups could perhaps research how oil companies could use logarithms – building on the context of video clip "1.09 Powers - Production Functions in Gold Mining and Oil Production" – and prepare a mini Powerpoint presentation to other groups. Students could prepare a multimedia resource that could be used for 'peer teaching' setting out the meaning of logarithms and their practical use by economists around the world.

It might be easier and more efficient for larger groups to focus on the video clips and then work through the activities and exercises.

Links with the online question bank

The online question bank does not include any no explicit or 'stand alone' questions but related questions involving indices can be found at:

http://www.metalproject.co.uk/METAL/Resources/Question_bank/Algebra/index.html

The on-line bank also includes elasticity questions at:

http://www.metalproject.co.uk/METAL/Resources/Question_bank/Economics%20applications/index.html

which can be extended to logarithms. This could be an effective strategy for further differentiating material.

Video clips

See 6.2 and 6.3 above for guidance. Clips can be found at:

http://www.metalproject.co.uk/Resources/Films/Mathematical_review/index.html and also include a further clip on population decline (1.08) and a effective summary film "1.10 Logarithms – Conclusion"

4. Discussion questions

1. What is the link between logarithms and powers/ indices?

2. What are logarithmic graphs and how they can help economists present data?

3. What is a 'natural logarithm' (ln or $\log_{e)}$ and how is this shown on a calculator?

5. Activities

ACTIVITY ONE

<u>Learning Objectives</u>

LO1: Students to understand the meaning of logarithms and understand rules

LO2: Students to be able to confidently manipulate simple expressions with logarithms

Task One

An economist discovers a link between inflation (I) and employment (E) which she believes to be:

$I = \ln(E^{\alpha})$

Complete the table and graph the relationship

α	E	E^{α}	$I = \ln(E^{\alpha})$
2	1		
2	1.1		
2	1.2		
2	1.3		
2	1.4		
2	1.5		
2	1.6		
2	1.7		
2	1.8		
2	1.9		
2	2		
2	2.1		
2	2.2		
2	2.3		
2	2.4		
2	2.5		
2	2.6		
2	2.7		
2	2.8		
2	2.9		
2	3		

Task Two

If: $I = 50 \times \alpha^{\beta}$ and $\alpha = 2.0$ and $I = 3.0$

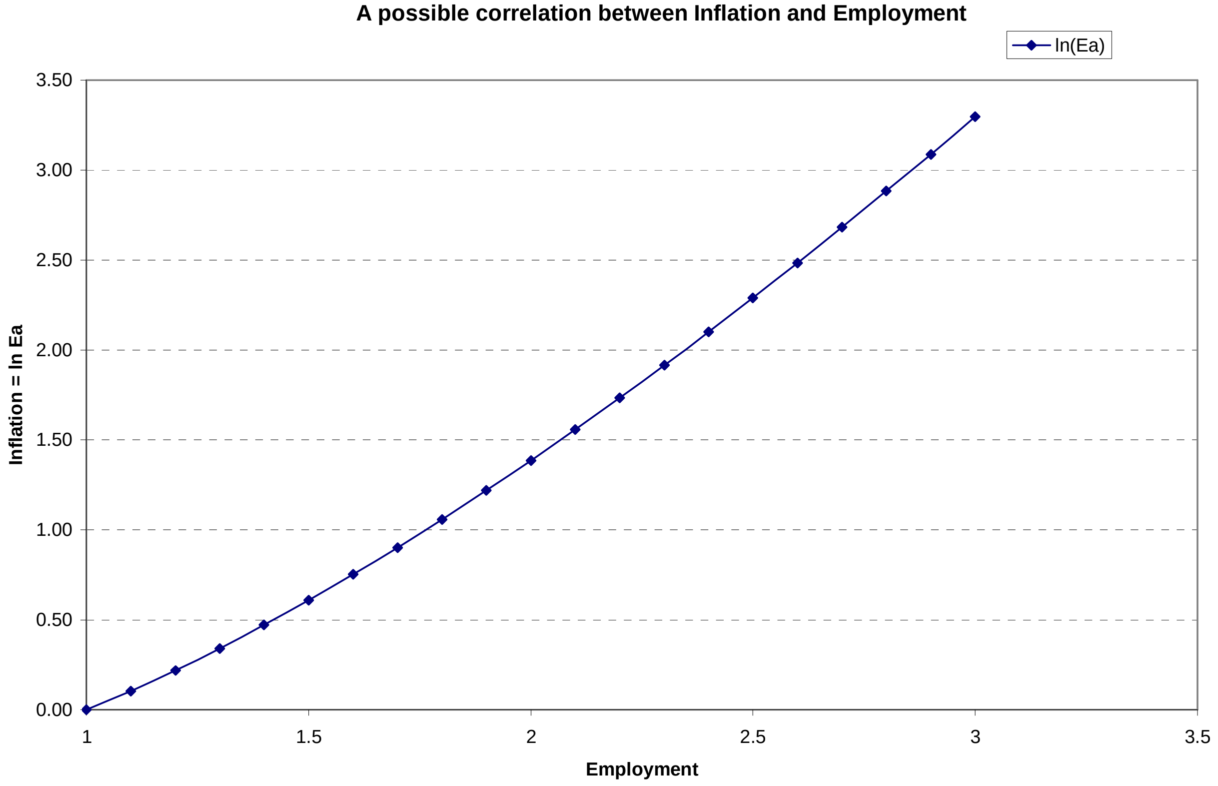
What is β?

ANSWERS

Task One

α	E	Eα	$I = \ln(E^{\alpha})$
2	1	1.00	0.00
2	1.1	1.11	0.10

2	1.2	1.24	0.22
2	1.3	1.41	0.34
2	1.4	1.60	0.47
2	1.5	1.84	0.61
2	1.6	2.12	0.75
2	1.7	2.46	0.90
2	1.8	2.88	1.06
2	1.9	3.39	1.22
2	2	4.00	1.39
2	2.1	4.75	1.56
2	2.2	5.67	1.73
2	2.3	6.79	1.92
2	2.4	8.18	2.10
2	2.5	9.88	2.29
2	2.6	11.99	2.48
2	2.7	14.61	2.68
2	2.8	17.87	2.88
2	2.9	21.93	3.09
2	3	27.00	3.30



Task Two

β= ln0.06/ln2 = -4.05 (to 2dp)

ACTIVITY TWO

<u>Learning Objectives</u>

LO1: Students to be able to explain the meaning and significance of logarithms

Task One

Complete the missing gaps using the words and phrases provided. There are more words and phrases than you need to correctly complete the gaps so select carefully!

Logarithms in economics

Logarithms are a useful economic tool, which are closely related to powers and **………..**
We know that 16 = $2^{\cdots}$ where the number **……….** is the ……………… or exponent. It is sometimes also known as the index.
Logarithms are particularly useful when analysing rates of ……………. and …………….. A pharmaceutical company, for example, might want to model rates of growth of …………………. or an economist might be interested to see how ……………. changes over time.
Problems concerning how much interest an investor can expect to receive or how much a sum of money would be worth in …………………. after a period of inflation are essentially issues surrounding ……………………….. These can be easily solved using logarithms.

In a simple logarithmic expression such as $A=B^{\alpha}$ w e can rearrange using logarithms to show that lnA=……..ln…….. where ln is the natural logarithm. A natural logarithm simply means a logarithm to the base ……………. where e is a constant approximately equal to ……..

Table of words

Indices	**compounding**	**growth**	**sub-divide**	**4**	**α**
2	**multiply**	**real terms**	**power**	**population**	**2.718**
nominal	**bacteria**	**e**	**change**	**B**	**argand**

ANSWERS

Task One

Logarithms are a useful economic tool, which are closely related to powers and **indices**.

We know that 16 = 2^4 where the number **4** is the **power** or exponent. It is sometimes also known as the index.

Logarithms are particularly useful when analysing rates of **change** and **growth**. A pharmaceutical company, for example, might want to model rates of growth of **bacteria** or an economist might be interested to see how **population** changes over time.

Problems concerning how much interest an investor can expect to receive or how much a sum of money would be worth in **real terms** after a period of inflation are essentially issues surrounding **compounding**. These can be easily solved using logarithms.

In a simple logarithmic expression such as $A=B^{\alpha}$ we can rearrange using logarithms to show that $\ln A = \alpha \ln B$ where ln is the natural logarithm. A natural logarithm simply means a logarithm to the base **e** where e is a constant approximately equal to **2.718**.

6. Top Tips

Students will probably need plenty of practice manipulating or rearranging logarithmic expressions. Perhaps more importantly students will benefit from a clear practical overview of why economists bother with logarithms! Again, the better the context, the more convincing the exposition and the higher the likelihood students will be able to understand their practical application.

7. Conclusion

Logarithms are a ‘higher level’ concept which will require careful introduction. Students should be given time to understand the concept, to understand the application and also to practise using logarithms to solve economic problems. The online question bank material could be adapted to create extended problems sets. Questions relating to price elasticity of supply and demand and production functions are obvious contexts.

Section 7: Basic rules of algebra

1. The concept of algebra

The concept of algebra will be familiar to many students. Rather like percentages, most will feel they know something, many will feel they could do algebra and perhaps many less will be able to use algebra accurately and confidently. The challenge then is to:

- show that algebra is valid and relevant;

- demonstrate how algebra can be used;

- provide opportunities for students to apply algebra; and

- build students' knowledge, competency and confidence in using algebra independently to solve economic and business problems.

2. Presenting the concept of algebra

The key to making an effective introduction and presentation of algebra is to convince the student audience that algebra is **relevant**. Students will tend to see algebra as something lofty or abstract and which is 'hard'. Of course, claiming something has impenetrable difficulty is sometimes a convenient way for students to avoid attempting to learn. It can also disguise a lack of learner confidence and a stronger desire to avoid failure and being 'shown up'.

Relevant and topical examples where algebra is used in day-to-day life include:

- air traffic control computers calculating arrival times and distances of aeroplanes;

- pit stop strategy for Formula 1 cars;

- working out the best time to sell a second-hand car;

- deciding when to re-order stock and how much e.g. a bakery ordering flour using Just-in-time production;

- estimating how much money a water company loses through leaky pipes;

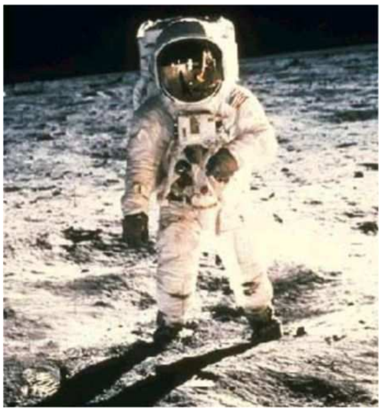
- working out how much it will cost to transport fruit (how many spherical oranges can be packed into a cuboid lorry?); and

- creating realistic forecasts of costs and revenue to arrive at break-even points.

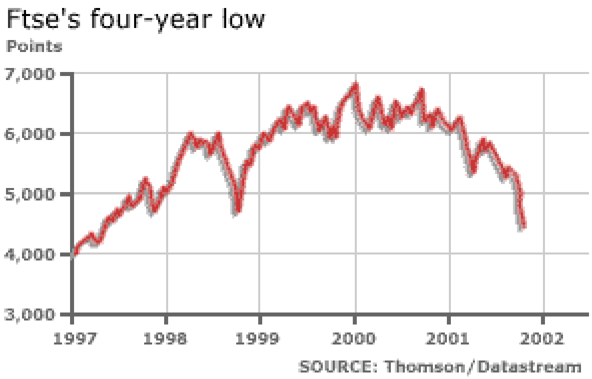
An 'algebra ice-breaker' could be to encourage students to work in pairs on researching "Algebra in Practice" by using the Internet. The task could be to simply find 10 examples where algebra is used in practice and students could produce a collage or poster to record and share their findings.

Alternatively, students could be given various images and asked to consider how algebra could be used. For example, a simple worksheet might look like this, with each image acting as a prompt for students to think, reflect or simply to create a springboard for students to undertake research. Some possible answers are shown underneath each image.

Calculating fuel load for long distance flight



Setting the trajectory for Apollo 11 to return safely



Working out the financial return on a portfolio of shares

3. Delivering the concept of algebra to small or larger groups

Students might find that simple exercises involving rearranging algebraic expressions would be useful: most students will probably find that the concept of algebra is straightforward but might need help with the rules or 'laws' of algebraic manipulation.

Two strategies might be useful here. First, smaller groups of students might find it useful to work with laminated cards with algebraic terms and work in pairs on constructing algebraic expressions provide by the lecturer.

e.g. using letters, numbers, powers can students construct an algebraic expression to represent the circular flow of income [$Y \equiv C+I+G+(X-M)$] and rearrange given information on key variables such as a linear consumption function $C=\alpha+\beta Y$ to arrive at:

$Y = [1/(1-b)] \times I + G\ (X-M)$.

Links with the online question bank

The online question bank covers an array of algebraic questions which include logarithms, powers and indices. These will help students to consolidate their knowledge, understanding and learning. Colleagues will want to explore:

http://www.metalproject.co.uk/METAL/Resources/Question_bank/Algebra/index.html. Perhaps one of the most areas section involves rearranging equations – a technique which many students can struggle with – and can be found at:

http://www.metalproject.co.uk/METAL/Resources/Question_bank/Economics%20applications/index.html

Video clips

Video clips can be found at:

http://www.metalproject.co.uk/Resources/Films/Mathematical_review/index.html.

Video clips 1.11 and 1.12 give simple but effective illustrations of how algebra can be used in a macroeconomic context. These clips could be used at the beginning of a module to create a good and contextual introduction to algebra.

4. Discussion questions

Algebra is, of course, a wide area of content. Discussion questions could centre on how algebra could be used to solve economic problems. Students could be encouraged to undertake independent research to demonstrate ways in which algebra might be used in the world of contemporary economics and business e.g. how is the FTSE-100 constructed? What expression would calculate the RPI? Which algebraic statement would express the net income of an employee (this could be linked to tax liability data from HM Customs).

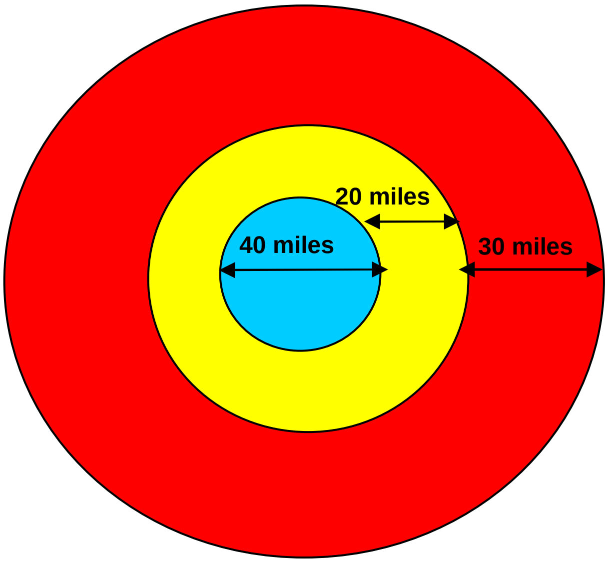
5. Activities

ACTIVITY ONE

Learning Objectives

LO1: Students to learn how to use simple algebraic formulae

An international space agency is planning to land a robot on Mars. The scientists produce a simple map of Mars and identify 3 zones - red, yellow and blue – upon which the robot could successfully land.



The zones are drawn as concentric circles. The width of the red zone is 30 miles, yellow zone 20 miles and the blue zone has a diameter of 40 miles.

Task One

Calculate the total area of the three zones

Task Two

Calculate the area of the red, blue and yellow zones.

ANSWERS

Task One

$A = \prod r^2 = 3.141 \times 70^2$

Area = 15393.8 miles^2

Task Two

Total	blue	yellow	red
15393.80	1256.637	3769.911	10367.26

ACTIVITY TWO

Learning Objectives

LO1: Students learn how to independently create a formula

LO2: Students learn how to apply their formula to solve a simple problem

Students can work in pairs to discuss how best to represent Claire's annual income

Task One

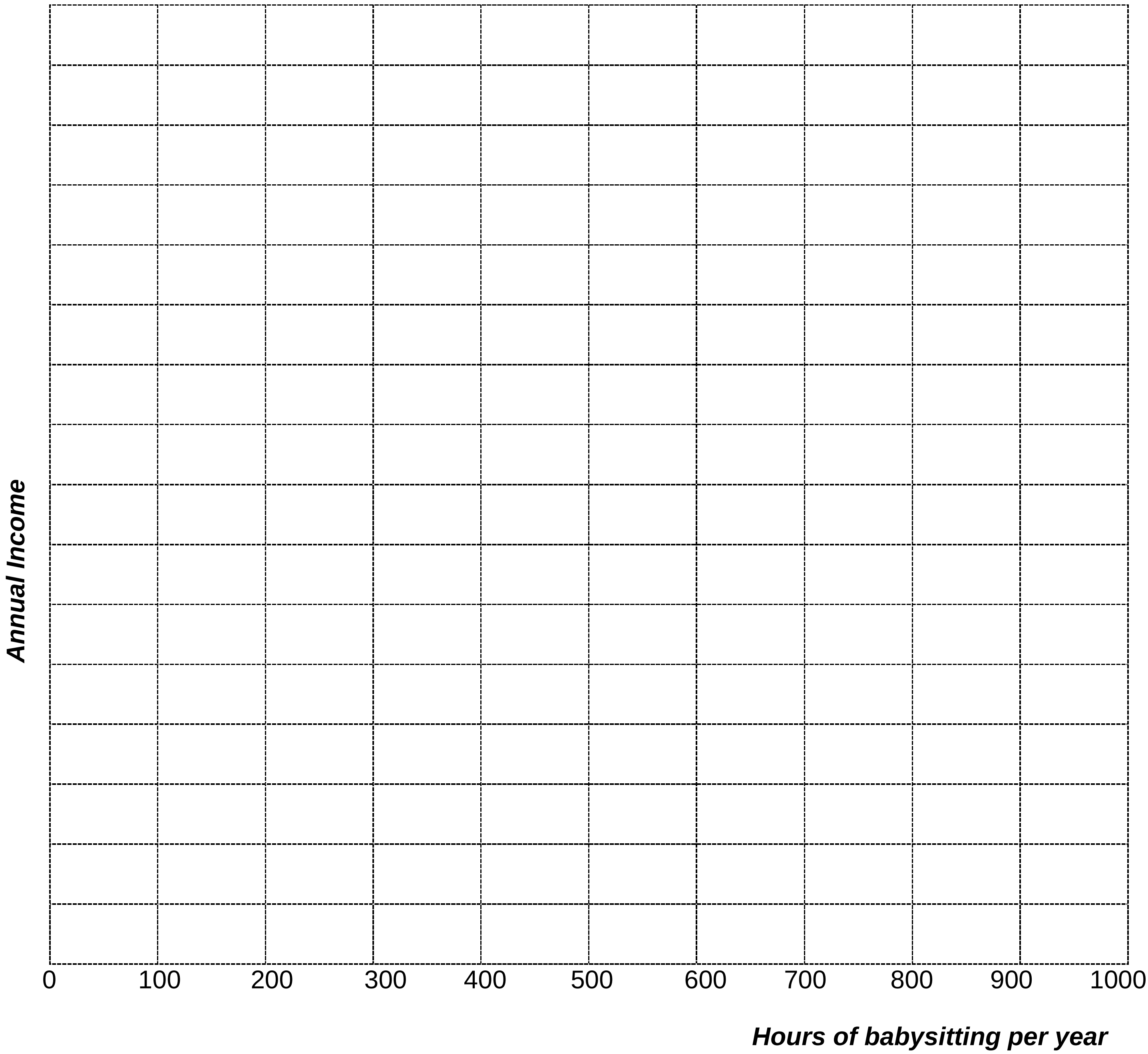
We are told that Claire's income is determined by three separate activities: her salary of £35,000, her part-time babysitting for which she charges £8.00 per hour and the small income she receives from the Government as a grant for her child. The grant is worth £5000 per year. Create a simple algebraic formula which could be used to calculate her total annual income

Task Two

Graph the income for Claire on the simple graph below. You will need to first:

i) label **both axes**; and

ii) consider a sensible scale for the vertical or y-axis.



Task Three

Claire's total income is taxed at 22%. Write a simple algebraic expression for her post-tax income.

Task Four

(a) Claire is delighted to be told she will receive a **salary rise** of 5% next year. What will her post-tax income formula be now?

(b) If Claire is told she will receive a 5% salary rise every year for *h* years, what will the new formula for her post-tax income be?

ANSWERS

Task One

Let: Y = Claire's total annual income

S = Claire's annual salary

B = Claire's income from babysitting

G = Claire's Government grant

n = the number of hours Claire babysits in a year

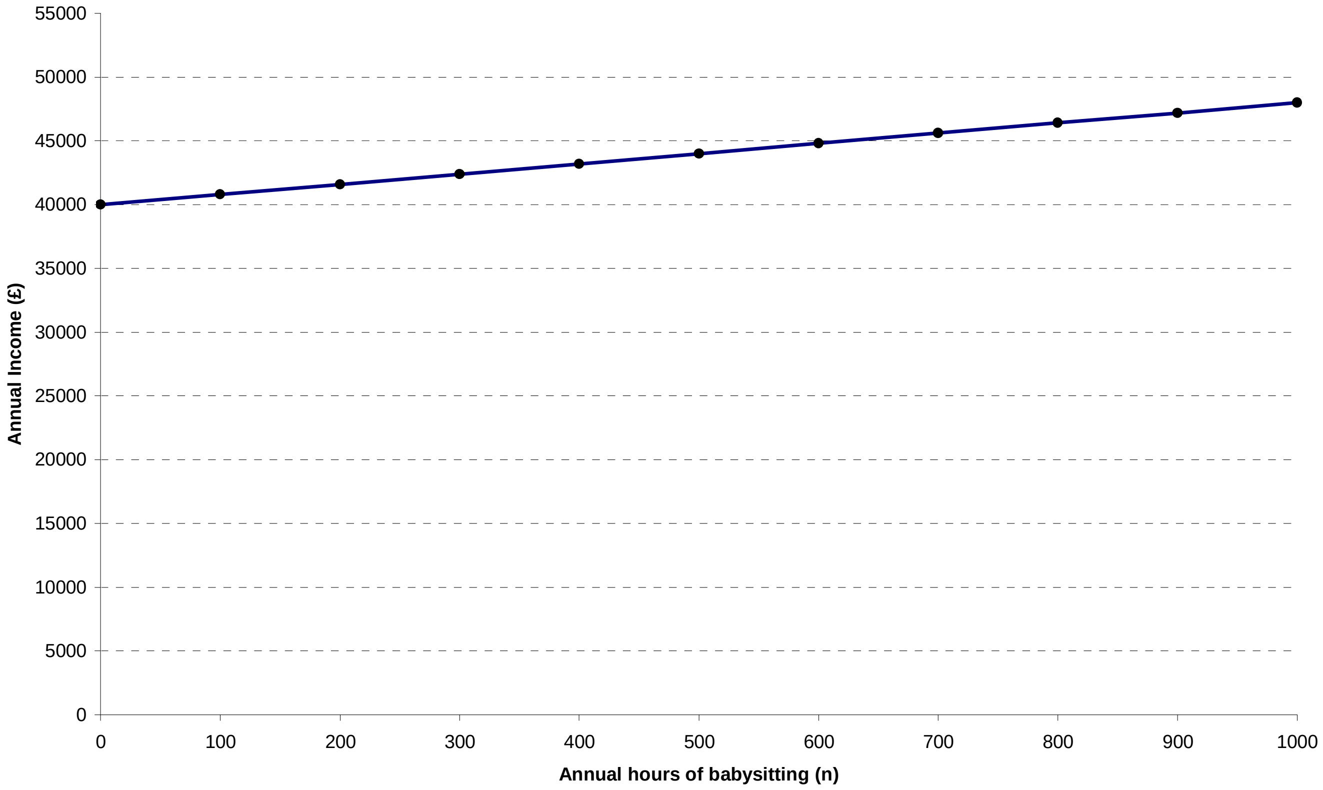
Then: Y = S + B + G

Y = £35000 + 8n + £5000

Y = £40000 + 8n or,

Y = 8(£5000 + n)

Task Two



Task Three

Y = 0.78(40000 + 8n)

Y = 31200 + 6.24n

Task Four

(a) Y = (1.05 x 31200) + 6.24n

Y = 32760 + 6.24n

(b) Y = (31200 x 1.05^h)+ 6.24n

6. Top Tips

1. Start the algebra topic at quite a basic level to maximise student access. Explain to students that algebra is the central element of an economist's toolkit and they need to be able to **use** it rather than simply **learn** it.

2. Give lots of practical examples where algebra is used to improve students' contextual understanding.

3. Provide opportunities for students to work together, perhaps by pairing a more able student with another who is less able or confident. Both will benefit from peer working and peer assessment.

4. Don't assume that students can use algebra because they have passed a Level 2 qualification in Maths: many students will have been 'taught to the test' and may not have an intuitive grasp of algebra and how it 'works'. Independent application may be quite difficult for many students.

7. Conclusion

Students need to learn:

- the rules of algebra;
- the significance of algebra in solving economic problems; and
- how to apply the rules of algebra to make sense of economic problems.

The more vocational or 'applied' the context of this learning, the more likely students will be able to access algebra as an economic tool rather than an abstract mathematical technique.